Empirical adaptive Galerkin FEM for parametric PDEs

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Adaptive stochastic Galerkin FEM (ASGFEM) with residual based a posteriori error estimation have shown to exhibit optimal convergence in practice for some standard parametric PDEs. However, their implementation is rather involved and requires significant effort when different problems should be tackled.

Motivated by recent results with empirical low-rank tensor regression in the framework of statistical learning, we examine a non-intrusive reconstruction method that only uses samples of the solution and yields the Galerkin projection with high probability. This can be seen as an easy to apply generalization of deterministic ASGFEM. For the sum of error and estimator, the proposed adaptive algorithm can be shown to converge.

To realize the error estimator, a sufficiently accurate tensor representation of the coefficient is required, which easily becomes challenging for instance when it is defined as an exponential function. We consider this common case and recall that it corresponds to the solution of a differential equation. It hence can be computed by means of a Petrov-Galerkin method for which error estimators are presented.

Joint work with: Nando Farchmin (PTB), Philipp Trunscke (Centrale Nantes).

References

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