Identifying the non-trivial zeros of the Riemann zeta function for prime counting function approximation in the Loewner framework

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The Loewner matrix pencil is an essential component of the realization and model reduction method for dynamical systems, originally proposed in [2] (known as the Loewner framework). This method was initially derived to construct reduced-order models (ROMs) of linear dynamical systems from data. More precisely, the transfer function of the ROM interpolates the original input-output data set corresponding to samples of a (rational) transfer function, or even of a complex irrational function (by enforcing rational approximation). Here, we are interested in the latter interpretation. We first describe a computationally and numerically simple procedure to estimate the "non-trivial" (harmonic) zeros of the famous Riemann ζ ("zeta") function (based on the Loewner framework); preliminary results are shown in Fig. 1. These approximated zeros are then used to recover the corrected Riemann prime counting function [1], approximating the prime number cardinality. We illustrate how efficient the Loewner framework is to recover this specific stair shape function.

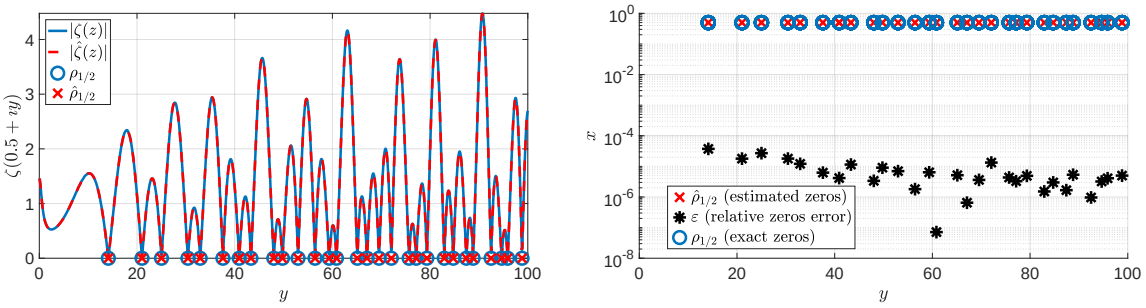

Figure 1: Left: comparison of the original and approximated functions for $y \in [0, 100]$. The zeros of each functions respectively are $\rho_{1/2}$ (blue circles) and $\hat{\rho}_{1/2}$ (red crosses). Right: exact (blue rounds) and estimated (red crosses) non-trivial complex zeros of the ζ function (x: real part, y: imaginary part). Relative point-wise mismatch error (black stars).

References

[1] B. Riemann. On the Number of Prime Numbers less than a Given Quantity (in German). *Monatsberichte der Berliner Akademie*, 1859.

[2] A. J. Mayo, A. C. Antoulas. A framework for the solution of the generalized realization problem. *Linear Algebra and its Applications*, vol.425, no. 2, pages 634–662, 2007.