Lower bounds in rational approximation to delays

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It is well-known that transfer functions of delay systems are hard to approximate by rational functions. This phenomenon, which is interesting from the point of view of approximation theory, is also a concern in problems where the frequency response of a device must be approximated by a model involving rational elements; *e.g.* in identification and matching. For instance, the transfer function $s \mapsto \exp\{-\tau s\}$ of a pure delay τ, where the complex variable s ranges over the right half-plane, has best rational approximant 0 on the imaginary axis: no nonzero rational function r exists such that $\|\exp^{-\tau\cdot} - r\|_{L^\infty(i\mathbf{R})} < 1$; here, $L^\infty(i\mathbf{R})$ refers to the *sup* norm on the imaginary axis, which represents the frequency axis in a system-theoretic context. We shall discuss a band-limited and norm-constrained version of the issue just mentioned, namely:

given an interval $I_0 = [-\omega_0, \omega_0]$ and $M > 0$, how small can the maximum of $|e^{-i\tau\omega} - r_n(i\omega))|$ be for r_n a rational function of type (n, n) (the ratio of two polynomials of degree at most n) wand $\omega \in I_0$, under the constraint that $\|e^{-i\tau\omega} - r_n(i\omega)\|_{L^\infty(\mathbf{R}\setminus I_0)} \leq M$?

Specifically, we will offer a lowerbound for this quantity.

The path we go is by perturbation of $s \mapsto \exp\{-\tau s\}$ into a Blaschke product and then comparison with L^2-approximation, using results from [1] which depend on a topological argument.

References

[1] L. Baratchart, S. Chevillard, T. Qian. Minimax principle and lower bounds in H2 rational approximation. *Journal of Approximation Theory*, 206:17–47, 2016.