Planar Polynomial PH Curves revisited

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A polynomial Pythagorean-hodograph (PH) curve has the property that its parametric speed — i.e., the derivative of the arc length with respect to the curve parameter — is a polynomial rather than the square root of a polynomial. Many computational advantages derive from this property and are useful in offsets, path planning, geometric design and similar applications.

In this talk, a geometric characterization for planar polynomial PH curves is presented. It is based on a variant of the dual representation of planar curves, where a curve may be regarded as the envelope of its tangent lines. The approach used here is illustrated with many examples.

A comparison is made with the state-of-art method : three–stage procedure that transforms any differentiable plane curve $r(t)$ into a PH curve $\hat{r}(t)$ through the use of the conformal map $z \to z^2$. In this framework, the Pythagorean structure of the hodograph $\hat{r}'(t)$ is achieved through the complex variable model. The *a priori* implementation is done through an algebraic model.

In the technique presented here, the Pythagorean property of the hodograph is achieved by a suitable geometric model. Notorious results for cubic PH curves and quintic PH curves are generalized. This geometric characterization provides an alternative three–stage procedure of generating plane polynomial PH curves. This work contributes to a different explanation of the theory and the applied algorithms for planar PH curves. It can be developed in various other related topics.