Point cloud registration for algebraic varieties using Riemannian optimization

Florentin Goyens
Université Paris Dauphine
goyensflorentin@gmail.com

We consider the registration of point clouds, that is, task of finding a transformation such that two point clouds overlap. We assume that the target and source point clouds belong to the same algebraic variety (up to a change in coordinates). Our method does not assume that a point correspondence is given.

Consider two algebraic varieties $V_1, V_2 \subset \mathbb{R}^n$ of the same degree d. We assume that there exists $Q \in \mathrm{SO}(n)$ and $a \in \mathbb{R}^n$ which define a rigid transformation $\mathcal{T}\colon \mathbb{R}^n \to \mathbb{R}^n$ such that the varieties V_1 and V_2 overlap, that is, for all $x_1 \in V_1$, we have $\mathcal{T}(x_1) := Qx_1 + a \in V_2$. Let $M_1 \in \mathbb{R}^{n\times s_1}$ and $M_2 \in \mathbb{R}^{n\times s_2}$ be composed of respectively s_1 samples in V_1 and s_2 samples in V_2. Given M_1 and M_2, our task is to estimate $Q \in \mathrm{SO}(n)$ and $a \in \mathbb{R}^n$ which define the transformation $\mathcal{T}$.

Our approach uses the monomial map, which is known to be rank deficient for matrices whose columns belong to an algebraic variety [2]. We use a rank-minimization procedure inspired from [2] using the Grassmann manifold. This allows us to write the registration as a smooth optimization problem defined on Riemannian manifolds, which we solve using off-the-shelf algorithms from the Manopt toolbox [1].

We also propose a framework that is robust to noise in the initial data. That is, given two point clouds $\hat{M}_1$ and $\hat{M}_2$, which are noisy versions of M_1 and M_2 we aim to find a rigid transformation $\mathcal{T}$ such that $\mathcal{T}(M_1)$ and M_2 overlap. To achieve this, we first attempt to recover M_1 and M_2 from $\hat{M}_1$ and $\hat{M}_2$, by solving a denoising problem, which is also formulated as a smooth Riemannian optimization problem in the manner of [2]. For the noisy and noiseless case, we show numerical results on synthetic examples which illustrate the efficiency and accuracy of our approach, see Figure 1.

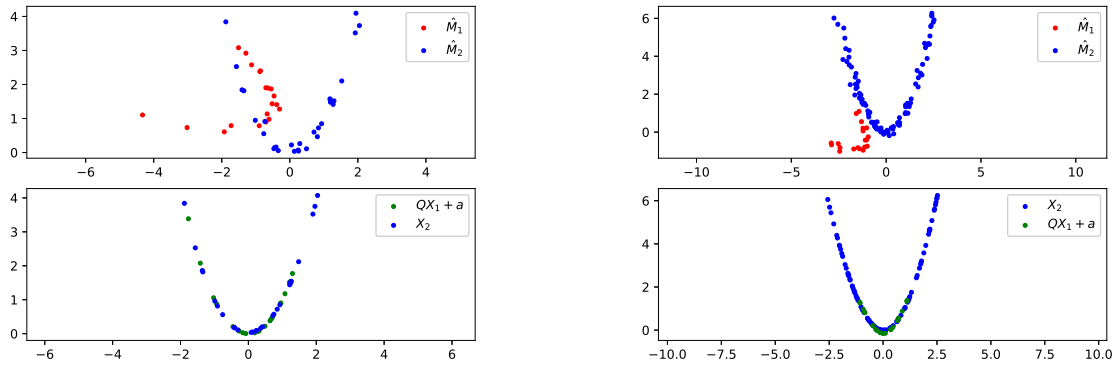

Figure 1: Noisy rigid registration of quadratic curves for overlapping and partially overlapping data sets. Input in the top image and output in the bottom image.

Joint work with: Coralia Cartis, Stéphane Chrétien.

References

[1] Boumal, N. and Mishra, B. and Absil, P.-A. and Sepulchre, R. Manopt, a Matlab Toolbox for Optimization on Manifolds, Journal of Machine Learning Research, 2014, http://www.manopt.org.

[2] Goyens, Florentin and Cartis, Coralia and Eftekhari, Armin. arXiv preprint arXiv:2109.06095, Nonlinear matrix recovery using optimization on the Grassmann manifold, 2021