Fast Formation of Matrices for Least-Squares Fitting by Tensor-Product Spline Surfaces

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The task of creating mathematical representations of free-form surfaces from scattered data is an important and therefore well-studied problem [1], which is particularly challenging and useful in industrial applications [2, 3]. Least-square fitting by tensor-product spline surfaces is a well-established method for approximating unstructured data, and is widely used in industry.

However, assembling the system of equations via the straightforward approach can be quite time–consuming. We propose to accelerate this process by employing the technique of sum factorization, which is frequently used in the context of isogeometric analysis, [4, 5].

Our approach consists of two steps. First, we introduce a regular grid onto which the parameters of the unstructured data are projected. Consequently, the expressions of the matrix entries can be rewritten in the structure of nested sums using the tensor-product and grid structure. This form admits the use of sum factorization, which is then employed in the second step. This novel approach has the potential of significantly reducing the computational effort of the matrix assembly, which is a substantial part of the overall computation time.

In this presentation, we provide estimates on the complexity of both the matrix assembly via the straightforward approach and via our approach. Furthermore, we quantify the expected relative assembly cost of the new method with respect to the standard method. We give several examples, including an example involving industrial data, to demonstrate how the choice of the grid influences speed, precision, and quality of the results, and confirm the expected time savings of the proposed method.

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References

[1] G. Greiner and K. Hormann. Interpolating and approximating scattered 3D-data with hierarchical tensor product B-splines. *Surface Fitting and Multiresolution Methods*, pages 163–172, 1997.

[2] C. Bracco, C. Giannelli, D. Großmann, S. Imperatore, D. Mokriš and A. Sestini. THB-spline approximations for turbine blade design with local B-spline approximations. Accepted to *SEMA-SIMAI granted to MACMAS 2019*, Springer.

[3] G. Kiss, C. Giannelli, U. Zore, B. Jüttler, D. Großmann and J. Barner. Adaptive CAD model (re-) construction with THB-splines. *Graphical models*, 76(5):273–288, 2014.

[4] P. Antolin, A. Buffa, F. Calabrò, M. Martinelli and G. Sangalli. Efficient matrix computation for tensor-product isogeometric analysis: The use of sum factorization. *Computer Methods in Applied Mechanics and Engineering*, 285:817–828, 2015.

[5] M. Pan, B. Jüttler and F. Scholz. Efficient matrix computation for isogeometric discretizations with hierarchical B-splines in any dimension. *Computer Methods in Applied Mechanics and Engineering*, 388:114210, 2022.