Projective equivalences of rational algebraic space curves using differential invariants

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We present an alternative method to existing algorithms [1, 2] for computing the projective equivalences between two rational space curves. The method is inspired in [3], where torsion and curvature, two classical and well-known differential invariants of space curves, are used to compute the similarities between two given space rational curves. In our case, we produce two projective curvature-like invariants κ_1 and κ_2, that can be used to characterize the existence of projective equivalences. In more detail, given two rational curves C_1 and C_2 properly parametrized by p, q, we prove that C_1, C_2 are projectively equivalent if and only if

$$\kappa_1(p) = \kappa_1(q)(\varphi), \quad \kappa_2(p) = \kappa_2(q)(\varphi),$$

where φ is a Möbius transformation. Then we can detect projective equivalence by checking whether or not the gcd of the polynomials involved in the above equation has a Möbius-like factor. In the affirmative case φ is obtained, and from here the projective equivalence itself can be easily computed. After finding the gcd, we can also use polynomial system solving; but in that case the system is substantially smaller compared to other approaches, which leads to better timings. The method has been implemented in Maple. A full description of the algorithm and the ideas behind it can be found in [4].

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References

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