Low-rank approximation of least squares fitting with bivariate tensor-product splines

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MTU Aero Engines is Germany's leading engine manufacturer and an established global player in the industry. We engage in the design, development, manufacture and maintenance of aircraft engines in all thrust and power categories as well as of stationary gas turbines. These activities are supported by a broad spectrum of CAE (Computer-Aided Engineering) tools and processes. In particular, underlying the aerodynamic simulations is the in-house geometry generation software.

The last years have seen growing precision and availability of 3D scanning tools. Hand-in-hand with it grows also the interest of our engineers to use the scanned data as an input of the geometry generator and to convert them into high-quality NURBS surfaces suitable for further use. While considerable effort has been spent on improving this process (see, e.g., [1, 2, 3, 4]), high expectations of our users mean that there is always room for further improvement.

In this talk, I will show an early-stage work from this area. Assuming that the input data are a two-dimensional grid of scalars, one can write them in a matrix form. Instead of approximating them directly with a bivariate tensor-product function, a low-rank approximation of this matrix can be constructed in the form

$$Z \approx \sum_{r=1}^{s} t_r \mathbf{u}_r \otimes \mathbf{v}_r \ .$$

For each $\mathbf{u}_r$ and $\mathbf{v}_r$ a *univariate* least-squares approximation can be computed, thus obtaining vectors $\mathbf{d}_r$ and $\mathbf{e}_r$, respectively, of control points. These control points can be re-assembled into a matrix form

$$C = \sum_{r=1}^{s} t_r \mathbf{d}_r \otimes \mathbf{e}_r \ .$$

Taking C as control points of a *bivariate* spline function yields a good approximation of the original data Z.

This method is a generalization of an existing approach presented in [5]. However, there are two new contributions. First, using least-squares approximation instead of interpolation leaves more freedom in the choice of the data parametrization and fitting basis. Second, we prove that when s is equal to the rank of the data matrix Z, then the elements of C are in fact the control points of the bivariate least-squares approximation of Z. Additionally, this approach is generalized to weighted least-squares with separable weights, which is of advantage, e.g., when approximating functions in L^2-sense using quadrature rules.

Joint work with: David Großmann and Bert Jüttler.

References

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