A Stable Method for Discretizing Differential Operators on Curves and Surfaces Using the RBF-FD Method

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Partial Differential Equations (PDEs) on arbitrary surfaces arise in many applied and natural science models. A notable example of solving PDEs on static surfaces is image processing. Applications of PDEs on evolving surfaces occur in material science and fluid dynamics. Additionally the fields of biology and computer graphics have applications for PDEs on both static and evolving surfaces.
There are three main categories of methods for solving PDEs on arbitrary surfaces: the methods that rely (i) on parametrization, (ii) on an embedding, and (iii) on triangulation. Embedding-type methods are quite simple in that they are based on the discretization of standard $\mathbf{R^3}$ operators rather than curve or surface-specific operators. One of the most common embedding methods is the closest point method (CPM), [1]. The surface is enclosed inside a thick layer of nodes that belong to a dense three-dimensional grid. Each one of these nodes takes the function value of the one associated with their closest point to the surface, implicitly imposing that the normal derivatives at each node is null. Under that constraint, the surface Laplacian is equivalent to its $\mathbf{R^3}$ analog.
The Radial Basis Functions Orthogonal Gradients method (RBF-OGr) is another embedding method, and was introduced in [2]. It benefits from the meshfree character of RBFs, which gives the flexibility to represent complex geometries in any spatial dimension while providing a high order of accuracy. This method is different from the CPM in that its computational domain is the point cloud that defines the manifold, instead of being being a thick layer of nodes around the surface. Every computation is performed on the surface, including the constraints of having null normal derivatives.
The fast RBF-OGr method [3] uses the finite-difference based RBF method, instead of the global standard RBFs which gave rise to dense differentiation matrices, a limiting factor on the size of the point cloud representing the surface. However, going from the global to the local RBF method have introduced a few sources of instabilities in the process. In this presentation, we will address the different stability issues and provide solutions. We will illustrate the procedure with a number of interesting examples.

Joint work with: C. Piret, C. Jacobs.

References

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