Comparison of 2 PDE models for anisotropic non local interactions in 2D

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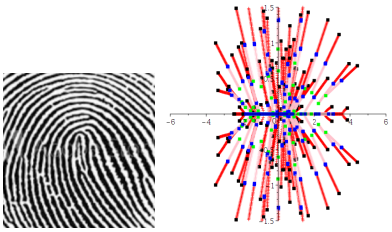
We compare model 1, developed by B. During et al. [1], aimed at studying fingerprint formation, and model 2 aimed at studying the dynamics of complex root sets of random polynomials under differentiation. [2]. The illustrations below show at left the ridges of a "stabilised" fingerprint formation. At right, the "trajectories" of the root set of a random polynomial P_n (represented by black solid boxes) and the root sets of several iterated derivatives of P_n.



Both phenomena are produced by anisotropic non local interactions in a bounded domain in 2D.
In both cases,

- When the number n of particles tends to infinity the sets of particles are represented by density functions, and asymptotic behaviors, are considered. They are created by balanced repulsion/attraction forces acting on initial configurations.
- The velocity of a particle, hence the motion of the set, is estimated from a mean field approximation.
- The anisotropy is represented by a local property, expressed by either a 2-tensor $T(x_j)$ or by an infinitesimal moving rectangle centered at x_j, (which can also be represented by a 2-tensor) in order to capture the stress created by the motion.

However,

- Model 1 was primarily designed for computing the stationary solutions of the PDE, whereas model 2 aims at understanding the begining of the motion, say for times t between 0 and 0.5, t is associated to $round(tn)$ derivations of P_n for $n >> 1$.
- The symbolic representations of the forces are different. Model 1 relies on explicit ensatz, which coefficients are quadratic combinations of exponentials of the distance between two roots. Model 2 relies on an electrostatic interpretation of the logarithmic derivative of P_n. The repulsion force is then computed by an intricate geometric construction.
- In model 1, a parameter χ must be chosen in advance for the definition of the tensor. In model 2 the variable c, which plays a similar role, is a function of the density, hence is time dependant.

And there are more differences ...

References

[1] B. During, C. Gottschlich, S. Huckemann, L. M. Kreusser, and C.-B. Schonlieb. *An Anisotropic Interaction Model for Simulating Fingerprints.* Journal of Mathematical Biology, 78 (2019), pp. 2171–2206.

[2] A. Galligo. *Modeling Complex Root Motion of Real Random Polynomials Under Differentiation.* hal 03577445 v1 (2022).