Computing with isometries and developable surfaces

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Developable surfaces constitute a prominent class of surfaces, besides being important for applications – they represent the shapes of thin sheet material as it bends from a flat state into space without stretching or tearing. Unfortunately, geometric modeling with developables is a notoriously difficult subject, and consequently there has been a great number of individual contributions to it. Nearly all of the well-known geometric properties of developables have been pressed into service for characterizing developability for different kinds of surface representations. These include global ones like the existence of an orthgonal network of geodesic curves, local ones like vanishing Gauss curvature, or the special geometry of tangent planes and rulings which developables are known to possess. Quite a few of these properties have led to effective computational treatments of developables, often by means of global optimization.

This presentation reports on some progress made in recent years. For example, both splines and meshes have been successfully used to model developables with curved creases [4, 1]. Very promising approaches to developability arise in connection with so-called checkerboard patterns which are associated to general quad meshes with regular combinatorics. One way to use them is to model developables not directly, but via isometric mappings from a planar domain. As it turns out, the possibility of modeling isometric mappings is highly useful in its own right [2, 3]. We do not believe that the checkerboard pattern method is exhausted yet, and in fact work on this topic is ongoing.

References

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