A use of the generalized Hopf fibration in minimal energy problems

Ujué Etayo
Universidad de Cantabria
etayomu@unican.es

During this talk we will explain how to make use of the internal geometric structure of the spheres of odd dimension to fairy distribute points in them.

Getting into the details: we will say that a set of points $\omega_N = \{x_1, \ldots, x_N\} \subset \mathbb{S}^d$ is well distributed if its associated discrete logarithmic or Riesz s-energy defined by

$$\mathcal{E}_{\log}(\omega_N) = -\sum_{i=1}^{N} \log ||x_i - x_j||, \qquad \mathcal{E}_s(\omega_N) = \sum_{i=1}^{N} \frac{1}{||x_i - x_j||^s} \tag{1}$$

is small. We will use well distributed points in $\mathbb{S}^2$ and $\mathbb{PC}^d$ and the generalized Hopf fibration $\mathbb{S}^1 \hookrightarrow \mathbb{S}^{2d+1} \to \mathbb{PC}^d$ to fairy distribute points in $\mathbb{S}^{2d+1}$. We use this technique with random point processes (determinantal point processes), see [1], for odd dimensional spheres and with a deterministic family of points: the so called Diamond ensemble, see [2].

Joint work with: Carlos Beltrán, Pablo García-Arce.

References

[1] Carlos Beltrán and Ujué Etayo. The Projective Ensemble and Distribution of Points in Odd-Dimensional Spheres. *Constructive Approximation*, 48(1):163–182, 2018.

[2] Carlos Beltrán and Ujué Etayo. The Diamond ensemble: A constructive set of spherical points with small logarithmic energy. *Journal of Complexity*, 59, 2020.