On the solution of a Riesz equilibrium problem and integral identities for special functions

Edward B. Saff

Vanderbilt University

`edward.b.saff@vanderbilt.edu`

ABSTRACT: We provide a full space quadratic external field extension of a classical result of Marcel Riesz for the equilibrium measure on a ball with respect to Riesz s-kernels. We address the case $s = d - 3$ for arbitrary dimension d, in particular the logarithmic kernel in dimension 3. The equilibrium measure for this full space external field problem turns out to be a radial arcsine distribution supported on a ball with a special radius. As a corollary, we obtain new integral identities involving special functions such as elliptic integrals and more generally hypergeometric functions. It seems that these identities are not found in the existing tables for series and integrals, and are not recognized by advanced mathematical software. Among other ingredients, our proofs involve the Euler – Lagrange variational characterization, the Funk – Hecke formula, the Weyl regularity lemma, the maximum principle, and special properties of hypergeometric functions.

Joint work with Djalil Chafai, Robert Womersley.