Dupin cyclide spline surfaces of arbitrary topology

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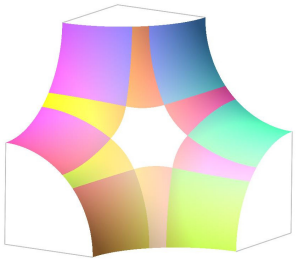
Regular circular quad-meshes with certain tangent data were used for smoothly blending principal Dupin cyclide patches [1]. Later this approach was extended in [2, 3] by allowing any $2n$-sided faces by filling them with virtually infinite rings composed of principal patches, when $n > 2$. Now we improve this approach: if all corner vertices of $2n$-sided hole are on a circle then this hole can be smoothly filled with the ring of $4n$ principal patches and one planar/spherical $2n$-sided patch in the middle. The method is demonstrated in the case of 3-beam corner in the left-side figure below.

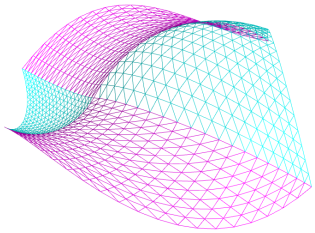
Smooth blending of triangular patches of Dupin cyclides along principal diagonal curves is investigated and expected to have more flexibility in modeling surfaces with arbitrary topology. In the case of cubic cyclides, the spline construction dual to Powell–Sabin elements [4] was extended by introducing foldings and branchings of the Gaussian map. In particular, this approach allows us to blend patches with different signs of curvature as well as to model monkey saddles (see middle and right figures below).

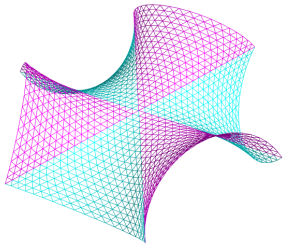
In order to track singularities Möbius classification of principal patches and principal diagonal curves was investigated. Furthermore, some topological restrictions on cyclide splines were derived when they are not containing planar or spherical patches:

- if the surface is closed (without boundary) then it is of torus topology;
- if the surface is simply connected and its boundary is composed of principal circles then the sum of its angles is the same as for a polygon with the same number of corners on a plane.

This proves that planar/spherical patches cannot be avoided in the hole-filling solution described above.







Joint work with: Rimvydas Krasauskas.

References

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