Optimization of a mutual shape based on the Fréchet-Nikodym metric for 3D shapes fusion

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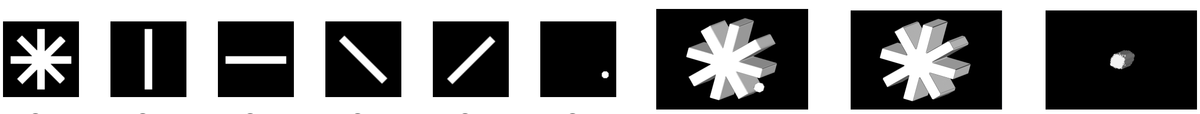
In the field of delineation of 2D or 3D regions of interest (ROI) in medical imaging, and especially due to the development of multimodal and multiparametric image acquisition devices, the combination of segmentations of anatomical structures from different sources is of interest. It is also essential to accurately evaluate the variability between experts delineation or algorithms with different parameters. In this work, we propose to estimate a mutual shape defined as the optimum of a statistical criterion based on information theory. Compared to our previous works [2, 3], we propose to interpret the mutual shape as a sum of distances of the Fréchet family and we extend our approach to 3D shapes fusion.

Let us consider $\{\Omega_1, ..., \Omega_n\}$, a set of n shapes corresponding to different segmentations of the same object. Shapes may include partial information or erroneous parts. Our goal is to compute a reference shape Ω_{ref} to estimate the considered object by combining the information given by all the regions Ω_i. In the literature, mean shapes are usually defined by minimizing the following functional: $J_1(\mu) = \sum_{i=1}^{n} d_1(\Omega_i, \Omega_{ref})$. The choice of the distance d is crucial and can lead to different results. For example, mean shapes can be computed by minimizing the sum of symmetric differences using $d_1(\Omega_i, \Omega_{ref}) = |\Omega_i \triangle \Omega_{ref}|$ where $\Omega_i \triangle \Omega_{ref} = (\Omega_i \cup \Omega_{ref}) \setminus (\Omega_i \cap \Omega_{ref})$.

Given a measure space (X, A, mes), a mutual shape can be defined more generally by minimizing the following functional: $J_2(\mu) = \sum_{i=1}^{n} d_2(\Omega_i, \Omega_{ref})$ where $d_2(\Omega_i, \Omega_{ref}) = mes(\Omega_i \triangle \Omega_{ref})$ is a finite signed measure of the symmetric difference not necessarily defined by the cardinality measure as in d_1. Under certain properties, such a distance is a shape metric known as the Fréchet-Nikodym distance [1]. This seems of interest because of the relevance of Fréchet distances in medical image analysis [4]. Such a distance can be considered under the umbrella of information theory. Indeed information theory quantities can be considered in terms of measure over sets. The joint entropy can be expressed using $H(D_i, T) = \text{mes}(\tilde{D_i} \cup \tilde{T})$ and the mutual information using $I(D_i, T) = \text{mes}(\tilde{D_i} \cap \tilde{T})$ where $\tilde{D_i}$ and $\tilde{T}$ are the abstract sets associated to Ω_i and Ω_{ref} (D_i and T define the random variables associated to the each characteristic functions). The mutual shape T is then defined by minimizing $E_2(T) = \sum_{i=1}^{n} (H(D_i, T) - I(D_i, T))$ expressed in a continuous framework and computed using shape optimization tools [2].

An example showing the interest of the combination of segmentation methods computed separately on four MRI modalities is given in [3]. In this current work, we extend our framework to 3D shapes fusion and we provide below a synthetic example to demonstrate the difference between the mutual shape (criterion J_2), the mean shape (criterion J_1) and the union of shapes. The algorithm is robust to the outlier Ω_6.



Ω_1 Ω_2 Ω_3 Ω_4 Ω_5 Ω_6 (a) Union (b) Mutual shape (c) Mean shape

Figure 1: (Ω_1 to Ω_6) : 2D Front views of 3D entries, (a) : Union of Ω_i (3D), (b) : Mutual Shape (3D), (c) : Mean shape (3D).

References

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