Symbolically separable low-dimensional nonlinear least squares

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Sparse interpolation, also called Prony's method or exponential analysis [1], consists in fitting a model of the form

$$\sum_{j=1}^{n} \alpha_j \exp(\phi_j t)$$

or

$$\sum_{j=1}^{n} \alpha_j t^{\phi_j}$$

to data y_k collected at sample points t_k for $k = 0, \ldots, N$ with $N \geq 2n - 1$.

A drawback of the method is the fact that the interpolation data need to be collected equidistantly.

Variable projection [2] applies to so-called separable problems, in which data are fitted by a linear combination of simple functions characterised by some nonlinear parameters, such as the above models. The nonlinear parameters are computed separately through optimisation and the linear coefficients are the solution of a least squares problem.

A drawback is that the method, when applied to higher-dimensional problems, easily gets stuck in a local minimum, unless one can supply a quite accurate starting point for the optimisation.

We consider some low-dimensional separable least squares problems of a Prony-like type, which offer the advantage that the objective function can be written down analytically, following ideas from [3].

Joint work with: Costanza Conti (University of Florence, Italy) and Annie Cuyt (University of Antwerp, Belgium).

References

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