A checkerboard pattern approach to isothermic surfaces

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A checkerboard pattern is a quadrilateral net with $\mathbb{Z}^2$ combinatorics where every second face is a parallelogram. Such a mesh can be easily obtained through midpoint subdivision from a general quadrilateral net. The structure of a checkerboard pattern is very suitable to describe discrete differential geometric properties of the net. In particular, we can use it to consistently define conjugate nets, principal curvature nets, a shape operator and Koenigs nets.

We find that the class of discrete principal curvature nets is invariant under Möbius transformations and can be studied in the projective model of Möbius geometry. Koenigs nets are exactly those nets that allow a discrete dualization. Analogously to the smooth case, they can be characterized by the existence of certain osculating conics (compare [1]) or by the equality of their Laplace invariants.

Isothermic nets can then be characterized as Koenigs nets that are also principal curvature nets. Again we can transform them using the Möbius transformation or dualization. The combination of both allows us to easily create examples of discrete minimal surfaces and their Goursat transformations.

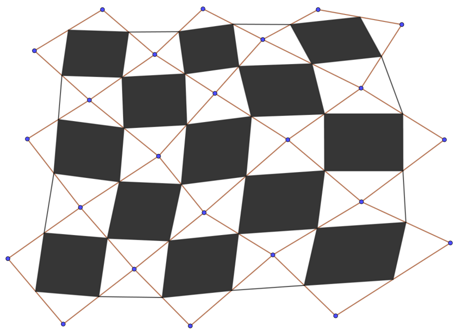

Figure 1: A checkerboard pattern created by midpoint subdivision. Every black face is a parallelogram.

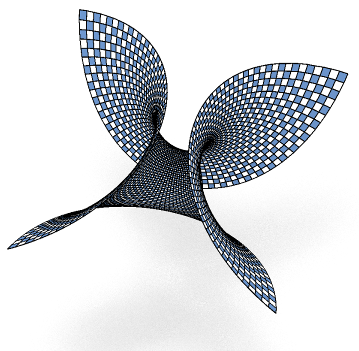

Figure 2: An isothermic checkerboard pattern on a discrete version of the Enepper surface.

References

[1] A. Doliwa. Geometric discretization of the Koenigs nets. *Journal of Mathematical Physics*, 44:2234-2249, 2003.