

Pythagorean-hodograph projections of spatial polynomial curves

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Our goal is to study whether spatial polynomial curve can be projected to a planar PH curve. This problem is motivated by [1], where the PH curves possessing projection to PH curve were constructed. By considering the set of all tangent directions to a given spatial curve C as the subset of projective plane \mathbb{P}^2 we obtain a planar curve T_C whose points correspond to the tangent directions to the original spatial curve. Since PH property is intimately related to the tangent behaviour of the curve it is not surprising that the it is encoded in the geometry of T_C . Indeed, with some additional assumptions, we show that C is PH curve if and only if T_C intersects the so called absolute conic in smooth points with even multiplicity. Similar geometric characterization can be formulated for planar curves as well. This approach enables to study the problem with the tools of projective geometry of planar curves. For example it can be easily seen that generic spatial cubic possesses exactly two orthogonal projections on PH curve, whereas curves of higher degrees cannot be projected in general. This motivates us to study oblique projections as well. Thus we will close by showing how spatial quintics can be projected in order to obtain PH curves.

Joint work with: Miroslav Lávička.

References

- [1] Rida T. Farouki and Marjeta Knez and Vito Vitrih and Emil Žagar. Planar projections of spatial Pythagorean-hodograph curves. *Computer Aided Geometric Design*, 91: 102049, 2021.