Quadrilateral mesh create from a given cross field

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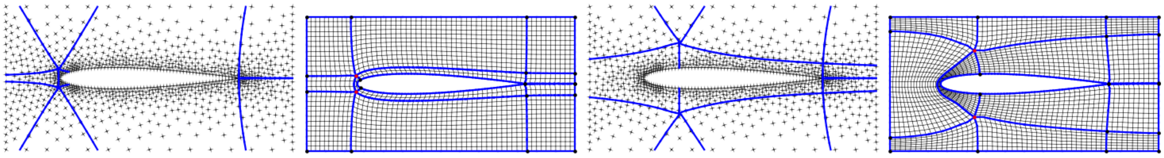

Figure 1: Representation of quadrilateral meshes from two different cross fields

Several high precision schemes with excellent quality and efficiency properties are built on quadrilateral meshes. However, the automatic generation of quadrilateral meshes with good quality elements is still a challenge. The method proposed in the article is of great interest with a final mesh structured by block, respecting the geometry and with good quality elements. However, it has some limitations, such as its unable to produce on very stretched geometries or on some domains without corners (for example, a 2D ring), a valid cross field which is the main notion on which the method is based. On the other hand, the variant of the Ginzburg-landau theory presented in [2] does not allow to deal with non-simply connected domains. In our work, we propose a new point of view allowing to solve the above mentioned limitations while keeping the structured aspect of the mesh and opening other possibilities on the generation of the cross field. To do this, our idea is to abstract, within the method, the generation of the cross-fields from the rest of partitioning process. We give ourselves a representation field which we then process in order to obtain a partitioning in blocks of 4 sides. We thus obtain different meshes according to the initial representation field.

More concretely, let Ω be a bounded domain and $\partial\Omega$ its boundary. We have $\Omega = \cup_i \Gamma_i$ when Ω is a non-simply connected domain and Γ_i, $\forall i$ denotes the connected components of $\partial\Omega$. We give ourselves a cross field u_c such that $deg(u_c, \partial\Omega) = deg(N_c, \partial\Omega)$ where N_c is the cross field associated with the normal of $\partial\Omega$, [2] and $deg(u_c, \partial\Omega)$, $deg(N_c, \partial\Omega)$ denote the Brouwer degrees of u_c and N_c on $\partial\Omega$ respectively. We then look for a field of angle ϕ on $\partial\Omega$ in order to align u_c on N_c by the rotation of angle ϕ of u_c. Our calculation of ϕ is inspired by the work presented in [2]. It consists in continuously propagating through the domain the angular difference between N_c and u_c which will allow to rectify the initial field and to align it on N_c. We strengthen this correction by introducing a new field w characterized by the formula $deg(w, \Gamma_i) = deg(N_c, \Gamma_i) - deg(u_c, \Gamma_i)$, $\forall i$. The introduction of this new field w is necessary when the initial cross field u_c does not respect the degree hypotesis presented above. This is especially the case when Ω is a non-simply connected domain where the Brouwer degree hypothesis is not necessarily respected on each connected component of $\partial\Omega$.

Our reformulation keeps the interesting property of producing structured block meshes while allowing to benefit from cross fields coming from the user. In our presentation, we will detail our reinterpretation and explain our argument with some algorithms, and then show the contribution of our point of view through several examples.

References

[1] N. Kowalski, F. Ledoux, and P. Frey, A PDE based approach to multi-domain partitionning and quadrilateral meshing, 2012, https://hal.sorbonne-universite.fr/hal-01076754

[2] R. Viertel and B. Osting, An Approach to Quad Meshing Based on Harmonic Cross-Valued Maps and the Ginzburg–Landau Theory, 2019, SIAM Journal on Scientific Computing