Conical surfaces

Jonas Tervooren
Technische Universität Wien
jtervooren@geometrie.tuwien.at.ac

The tangent planes along each parameter-line of a conjugate net, $f : \mathbb{R}^2 \supset U \to \mathbb{R}^3$, envelop a developable surface. We call a conjugate net *conical*, if the developable surfaces along one of the families of parameter-lines are cones or cylinders. Hence, one can easily approximate a given surface by tangential cone patches, if a conical parametrization is known. This has interesting applications for cladding of building facades, because conical strips are only curved in one direction and can be constructed by bending metal sheets or glass. The collection of cone patches, can be understood as a semi discrete surface itself, by considering the intersection curves of neighboring cones, which are a discrete family of smooth curves. We found a way to design semi-discrete conical surfaces by using NURBS curves.

Conical surfaces are not only of practical interest, but also exhibit a rich theory. Similar to isometric surfaces, which can be characterized by the existence of a transformation group (Darboux transformations), we proved that a conjugate net is conical if and only if there exist a special family of Combescure transformations, called *conical Combescure transformations* (CCT). Since every non zero function of one parameter defines a CCT, conical nets always appear as a families of parallel related nets. Further, conical nets are invariant under projective transformations of the ambient space. A special subclass of conical nets, containing the well known canal surfaces, is given by the orthogonal ones. We proved that an orthogonal net is conical if and only if every curve of one family of parameter-lines has constant geodesic curvature and consequently lies on a sphere. This observation enabled us to find an explicit construction for all orthogonal conical nets. Another interesting subclass are nets, that have tangential cones at both families of parameter-lines. We call them double conical nets and proved that a conical net is *double conical* if and only if it is a Koenigsnet. Applying a theorem of Darboux, gave us that every orthogonal double conical net is Möbius equivalent to a surface of revolution, cone or cylinder.

Finally, we found a discretization of conical nets as quadrilateral surfaces $f : \mathbb{Z}^2 \supset U \to \mathbb{R}^3$ and showed that most of the results for smooth conical nets stay true for their discrete counterparts. In the talk, we will give a general introduction to conical nets and then present the new results and constructions for both smooth and discrete conical surfaces.

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