Construction of G^2 Hermite interpolants with prescribed arc lengths

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Most methods to construct curves rely on the interpolation of discrete data, such as points, tangents or curvatures. If, in addition, we want to prescribe the total length of the resulting curve, in general we need to use iterative approximate methods. Pythagorean–hodograph (PH) curves are polynomial curves with the distinctive property of possessing arc lengths exactly determined by simple algebraic expressions in their coefficients. Hence the problem of constructing G^2 curves, that interpolate points, tangent directions and curvatures, and in addition have prescribed arc-length, can be exactly addressed. In this talk both the planar and the spatial problems are investigated considering PH curves of degree 7. For planar curves it is shown that in order to have a solution for any data, it is convenient to consider *biarcs*, keeping the degree to 7. In this case the solution of the G^2 continuity equations can be derived in a closed form, depending on four free parameters. By fixing two of them to zero, it is proven that the length constraint can be satisfied for any data. The proposed method is easy to implement and simple to use in practice, moreover, it can be directly applied to a (local) construction of G^2 continuous interpolating splines. Its extension to the spatial case is also possible and the main ideas behind this construction will be presented.

Joint work with: Marjeta Knez (University of Ljubljana) and Francesca Pelosi (University of Roma Tor Vergata).