Distance related problems in planar graphs and graphs on surfaces

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Let G be graph and let $d_G(u, v)$ denote the distance between vertices u and v of G. Three key values associated to the graph are

- the *Wiener index*, defined by $\sum_{u,v \in V(G)} d_G(u, v)$ and closely related to the *average distance* of G;
- the *sum of inverse geodesic lengths*, defined as $\sum_{u \neq v \in V(G)} \frac{1}{d_G(u,v)}$ and closely related to the *total efficiency* of G;
- the *diameter*, defined by $\max\{d_G(u, v) \mid u, v \in V(G)\}$.

All these values can be computed trivially by explicitly computing all the pairwise distances in G. Can we compute these values faster, without computing all the pairwise distances? Lower bounds assuming the Strong Exponential Time Hypothesis (SETH) were shown by Roditty and Vassilevska Williams [3]. Most interestingly, if the graph has n vertices and $\Theta(n)$ edges, no algorithm can compute those values in $O(n^{1.99999})$, assuming SETH.

I will discuss how these values can be computed in subquadratic, namely $\tilde{O}(n^{9/5})$ time, for n-vertex planar graphs and graphs on surfaces of constant genus. The main ideas are from [1, 2], but I will explain an alternative point of view that represents all distances in the graph a compact way.

References

[1] S. Cabello. Subquadratic algorithms for the diameter and the sum of pairwise distances in planar graphs. *ACM Transactions on Algorithms*, 2018.

[2] S. Cabello. Computing the inverse geodesic length in planar graphs and graphs of bounded treewidth. *ACM Transactions on Algorithms*, accepted

[3] L. Roditty and V. Vassilevska Williams. Fast approximation algorithms for the diameter and radius of sparse graphs. *Proc. 45th ACM Symposium on Theory of Computing*, STOC 2013.