Approximating Singular Measures on the Torus with Moment Polynomials

Paul Catala
Institute of Mathematics, University of Osnabrück
paul.catala@uos.de

We discuss polynomial approximations of nonnegative Radon measures supported on *arbitrary* domains of the d-dimensional torus, given moments up to some degree n. We introduce new estimates, that can be computed from the truncated moment matrices of the measures and provide complementary recovery guarantees.

Our first estimate for a measure μ is given by the convolution of μ with the Fejér kernel. A similar construction was also considered in [1]. The resulting trigonometric polynomial can be evaluated efficiently on a grid using only Fast Fourier Transforms. We establish sharp bounds on the rate of convergence of this proxy towards μ with respect to the Wasserstein-1 distance. Second, we introduce a certifying polynomial, that can identify exactly the Zariski closure of the support at finite degrees. This polynomial can be computed from the singular value decomposition of the moment matrix, and we show that it converges pointwisely towards the characteristic function of the support.

Joint work with: Mathias Hockmann, Stefan Kunis, Markus Wageringel.

References

[1] Mhaskar, H. Super-Resolution Meets Machine Learning: Approximation of Measures. *J. Fourier Anal. Appl., 25(2)*, pages 3104–3122, 2019.