Repulsive Curves and Surfaces

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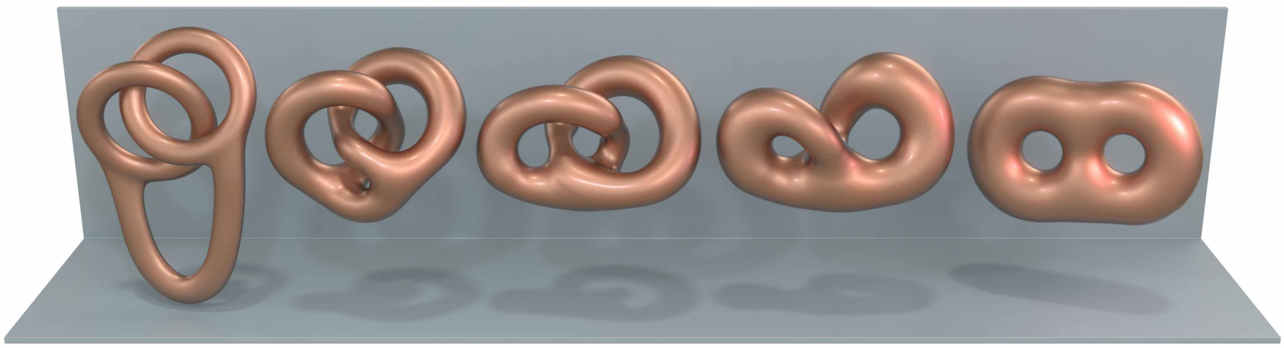

Functionals that penalize bending or stretching of a surface play a key role in geometric and scientific computing, but to date have ignored a very basic requirement: in many situations, surfaces must not pass through themselves or each other. This paper develops a numerical framework for optimization of surface geometry while avoiding (self-)collision. The starting point is the tangent-point energy, which effectively pushes apart pairs of points that are close in space but distant along the surface. We develop a discretization of this energy for triangle meshes, and introduce a novel acceleration scheme based on a fractional Sobolev inner product. In contrast to similar schemes developed for curves (cf. [2]), we avoid the complexity of building a multiresolution mesh hierarchy by decomposing our preconditioner into two ordinary Poisson equations, plus forward application of a fractional differential operator. We further accelerate this scheme via hierarchical approximation, and describe how to incorporate a variety of constraints (on area, volume, etc.). Finally, we explore how this machinery might be applied to problems in mathematical visualization, geometric modeling, and geometry processing.

Joint work with: Chris Yu, Caleb Brakensiek, Keenan Crane

References

[1] C. Yu, C. Brakensiek, H. Schumacher, and C. Keenan Repulsive Surfaces. *ACM Trans. Graph.*, 40(6):1–19, 2021.

[2] C. Yu, H. Schumacher, and C. Keenan Repulsive Curves. *ACM Trans. Graph.*, 40(2):1-21, 2021.