Bear subdivision schemes for modeling smooth surfaces

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We suggest a new class of stationary subdivision schemes based on matrix dilation for modeling smooth surfaces. In each iteration our scheme doubles the number of nodes, their new coordinates are weighted averages of old coordinates along one direction. This direction changes every iteration and never repeats. We propose a method for calculating the Holder regularity of limit surfaces based on the work [1] and program [2]. The results of these calculations are surprising and show that the regularity of such schemes in some cases exceeds the regularity of classical schemes of the same order.

Recall that one-dimensional schemes are defined by a finite set of the coefficients $\{c_k\}_{k\in\mathbb{Z}}$. A corresponding subdivision operator S maps a sequence $u \in \ell_\infty(\mathbb{Z})$ to the sequence $Su \in \ell_\infty(\mathbb{Z})$: $Su(k) = \sum_{j\in\mathbb{Z}} c_{k-2j} \cdot u(j)$. Let $f_0(\cdot) = u(\cdot)$ be an initial function ($\mathbb{Z} \to \mathbb{R}$) (control polygon). Then the next sequence f_1 is defined on $\mathbb{Z}/2$ as $f_1(\frac{1}{2}k) = Su(k)$. Repeating this process we get sequences f_i defined on $\mathbb{Z}/2^n$ which in case of the convergence tend to a limit curve (pointwise).

In the case of matrix dilation, the subdivision operator $S : \ell(\mathbb{Z}^2) \to \ell(\mathbb{Z}^2)$ is generalized as (see, for example, [3]) $Su(k) = \sum_{j\in\mathbb{Z}^2} c_{k-Mj} \cdot u(j)$, where M is an expanding matrix (i.e. all its eigenvalues $|\lambda_i| > 1$).

Our schemes exploit a Bear matrix $M_B = \begin{pmatrix} 1 & -2 \\ 1 & 0 \end{pmatrix}$. It is expanding with the property $m := |\det M| = 2$. It is known that there are three different (up to affine similarity) expanding integer matrices with $m := |\det M| = 2$, the Bear matrix is one of them.

The coefficients of our schemes are defined as follows

$$c(k_1, 0) = \frac{1}{2^{K-1}} \binom{K}{k_1}$$

for $k_1 = 0, 1, \ldots, K$, where K is a parameter. The scheme with $K = 4$ is illustrated on fig 1. All these schemes have a low complexity because of a small number of coefficients (all coefficients are from one line). Using the method based on the notion of joint spectral radius of transition matrices, we calculate the regularity of their limit surfaces for $K \le 5$. The most interesting result is that the scheme with $K = 3$ produces C^2 surfaces, the scheme with $K = 4$ produces C^3 surfaces, and the scheme with $K = 5$ produces C^4 surfaces which is better than for corresponding classical schemes with the same degree of th mask.

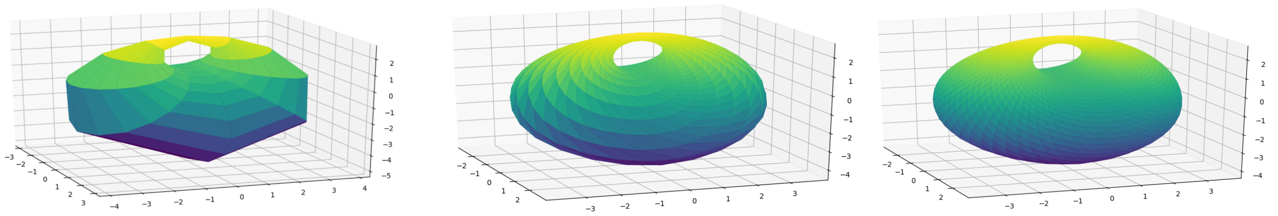

Figure 1: A Dupin cyclide generated by our scheme with $K = 4$: iterations 0, 2 and 4.

References

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