Provable Phase retrieval via Mirror descent

Jean-Jacques GODEME
Normandie Univ, ENSICAEN, CNRS, GREYC, France
jean-jacques.godeme@unicaen.fr

We consider the problem of phase retrieval, recovering an $n-$dimensional real vector from the magnitude of its $m-$ linear measurements. This paper presents a new approach [1] allowing to lift the classical global Lipschitz continuity requirement through the use of a non-euclidean Bregman divergence, to solve the nonconvex formulation of the phase retrieval problem [2]. We show that when the measurements are sufficiently large, with high probability we can recover the desired vector up to a global sign change. Our set-up uses careful initialization via a spectral method and refines it using the mirror descent with a backtracking procedure to find the optimal solution. We show local linear convergence with a rate and step-size independent of the dimension. Our results are stated for two types of measurements: those drawn independently from the standard Gaussian, and those obtained by Coded Diffraction Patterns (CDP) for Randomized Fourier Transform.

Problem Formulation: Our goal is to recover a vector $x \in \mathbb{R}^n$ from $y_r = |a_r^* x|^2, r \in \{0, 1, .., m\}$, where $a_r \in \mathbb{C}^n$ are the sensing vectors. We cast this as solving the following non-convex optimization problem: $\min_{x \in \mathbb{R}^n} f(x) = \frac{1}{4m} \sum_{r=1}^{m} \left(y_r - |a_r^* x|^2\right)^2$. Our recovery procedure starts at some vector x^0 via a spectral method, and then for $k = 1, 2, \ldots$, implements the iteration

$$x^k = (\nabla\psi)^{-1}\left(\nabla\psi(x^{k-1}) - \gamma_k \nabla f(x^{k-1})\right) \tag{Mirror Descent}$$

where $\psi(x) = \frac{1}{4}\|x\|^4 + \frac{1}{2}\|x\|^2$ is a kernel generating distance proposed in [3] and γ_k is the stepsize sequence either fixed in $]0, 1/(3+\delta)]$, $\delta > 0$, or adjusted via backtracking. ψ enjoys many desirable properties making the (Mirror Descent) scheme very efficient. Our main result is the following.

Theorem 0.1 *Let $\delta > 0$ sufficiently small. Suppose that the number of measurements m is large enough i.e $m \geq C(\delta) n \log(n)$ in the gaussian case (respectively $m \geq C(\delta) n \log(n)^3$ in the CDP case). If* (Mirror Descent) *is initialized with a spectral method, then with probability at least $1 - 10e^{-cn} - \frac{8}{n^2}$ ($c > 0$ is a fixed numerical constant) for the gaussian (respectively $1 - \frac{1}{n^3}$ for the CDP model) , we have*

$$\operatorname{dist}^2(x^k, x) = \mathcal{O}\left(\left(1 - \frac{1}{3(3+\delta)}\right)^k\right) \tag{1}$$

where $\forall z \in \mathbb{R}^n$, $\operatorname{dist}(z, x) = \min\{\|z - x\|, \|z + x\|\}$.

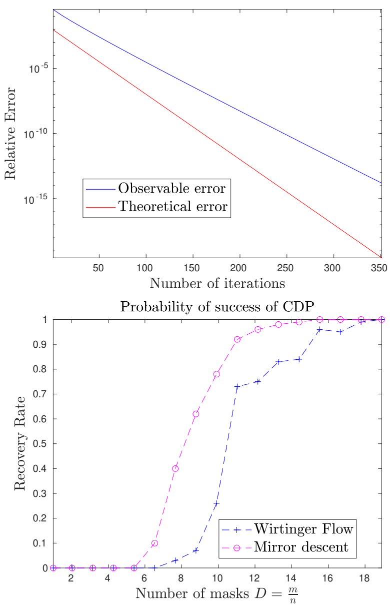

Figure 1: Example of relative error and phase transition

Joint work with: Jalal Fadili, Xavier Buet, Myriam Zerrad, Claude Amra and Michel Lequime.

References

[1] Jean-Jacques Godeme, Jalal Fadili, Xavier Buet, Michel Lequime, Claude Amra, "Provable Phase Retrieval via Mirror descent", 2021, (to be submitted).

[2] Emmanuel Candès, Xiaodong Li, and Mahdi Soltanolkotabi, "Phase Retrieval via Wirtinger Flow: Theory and Algorithms," IEEE Transactions on Information Theory, vol. 61, no. 4, pp. 1985-2007, 2015.

[3] Jérôme Bolte, Shoham Sabach, Marc Teboulle, and Yakov Vaisbourd, "First Order Methods beyond Convexity and Lipschitz Gradient Continuity with Applications to Quadratic Inverse Problems", SIAM Journal on Optimization, vol. 28, no. 3, pp. 2131?2151, 2018.