Gauss–Legendre polynomial for the shape control of parametric curves

Soo Hyun Kim
Applied Algebra and Optimization Research Center, Sungkyunkwan University, Korea
5hkim@skku.edu

The Gauss–Legendre polygon was developed [1, 2] as the rectifying control polygon of Pythagorean hodograph (PH) curves, whose length is the same as the arc length of the corresponding PH curve. We here focus on the polynomial basis for the Gauss–Legendre polygon. As a Bézier curve is expressed as the linear combination of the Bézier control points with the coefficients given by the Bernstein polynomials, a PH curve can be expressed as the linear combination of the Gauss-Legendre control points with the coefficients determined by some polynomials, which we call the Gauss–Legendre polynomials. We show that the Gauss–Legendre polynomials can be constructed from the Lagrange interpolator defined over the roots of the Legendre polynomials. We investigate various properties of the Gauss–Legendre polynomials including symmetry, partition of unity, critical points, derivatives, and integrals. We also analyze the pros and cons of this polynomial over other polynomials.

Joint work with: Hwan Pyo Moon, Song-Hwa Kwon.

References

[1] S.H. Kim, H.P. Moon. Rectifying control polygon for planar Pythagorean hodograph curves. *Computer Aided Geometric Design*, 54:1–14, 2017.

[2] S.H. Kim, H.P. Moon. Deformation of spatial septic Pythagorean hodograph curves using Gauss–Legendre polygon. *Computer Aided Geometric Desig*, 73:16–34, 2019.