Learning mean curvature flows with neural networks

Simon Masnou
Université Claude Bernard Lyon 1
masnou@math.univ-lyon1.fr

The mean curvature flow is an emblematic geometric flow which is very naturally related to various numerical and physical applications, e.g. in image / data processing or in material sciences. The talk will be devoted to new, efficient, and accurate numerical methods based on neural networks for the approximation of the mean curvature flow of either oriented or non-orientable surfaces [1]. To learn the correct interface evolution law, the neural networks are trained on phase field representations of exact evolving interfaces. The structure of the networks draws inspiration from splitting schemes used for the discretization of the Allen-Cahn equation. But when the latter approximates the mean curvature motion of oriented interfaces only, the proposed approach extends very naturally to the non-orientable case. In addition, although trained on smooth flows only, the proposed networks can handle singularities as well. Furthermore, they can be coupled easily with additional constraints. Various applications will be shown to illustrate the flexibility and efficiency of our approach: mean curvature flows with volume constraint, multiphase mean curvature flows, numerical approximation of Steiner trees, numerical approximation of minimal surfaces.

Joint work with: Elie Bretin, Roland Denis, Garry Terii.

References

[1] E. Bretin R. Denis S. Masnou G. Terii, Learning phase field mean curvature flows with neural networks, arXiv:2112.07343, 2021.