Lebesgue-type inequalities in greedy approximation with respect to bases

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Let $(\mathcal{G}_m)_{m=1}^{\infty}$ denote the thresholding greedy algorithm (TGA for short) of a basis of a Banach space $\mathbb{X}$. To measure the efficiency of the TGA is customary to use the Lebesgue parameters $(\boldsymbol{L}_m)_{m=1}^{\infty}$, defined for each $m \in \mathbb{N}$ as the optimal constant $C = C(m)$ such that

$$\|f - \mathcal{G}_m(f)\| \leq C \, \|f - g\|$$

for all $f \in \mathbb{X}$ and all linear combinations, g, of m vectors of the basis.

Calculating the exact value of the Lebesgue constants can be in general a difficult task, so in order to study the efficiency of non-greedy bases we must settle for obtaining easy-to-handle parameters that control the asymptotic growth of $(\boldsymbol{L}_m)_{m=1}^{\infty}$. Most of of such parameters and estimates have sprung from the celebrated characterization of greedy bases by Konyagin and Telmyakov [1]. In fact, several authors have obtained estimates for the Lebesgue constants, either of general bases or of bases with some special features, in terms of the unconditionality constants $(\boldsymbol{k}_m)_{m=1}^{\infty}$ and a sequence of democracy-like parameters that fits their purposes.

In this talk, we introduce a new sequence of democracy-like parameters, which we call $(\boldsymbol{\lambda}_m)_{m=1}^{\infty}$, which combined linearly with the unconditionality parameters determines the growth of the Lebesgue parameters. That is,

$$\boldsymbol{L}_m \approx \max\{\boldsymbol{k}_m, \boldsymbol{\lambda}_m\}, \quad m \in \mathbb{N}.$$

This result provides an answer to a problem raised by Temlyakov during the *Concentration week on greedy algorithms in Banach spaces and compressed sensing* held on 18–22 July, 2011, at Texas A&M University.

Joint work with: Fernando Albiac, Pablo Berná.

References

[1] S. L. Konyagin and V. N. Temlyakov. A remark on greedy approximation in Banach spaces. *East J. Approx.*, 5(3):365–379, 1999.