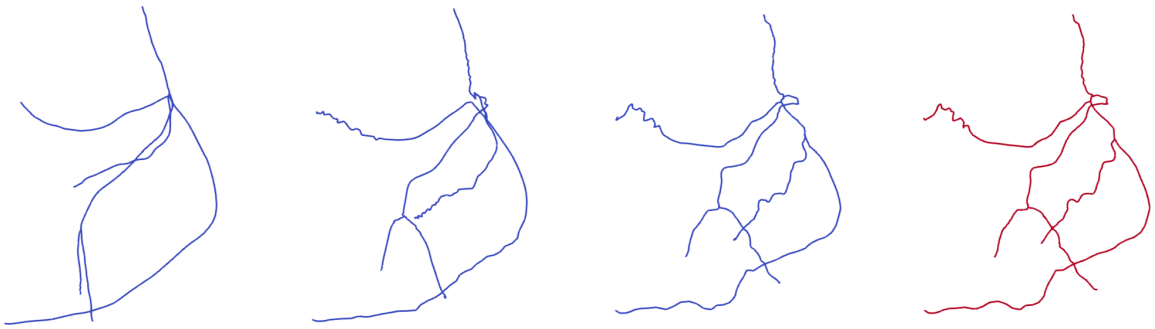
Diffeomorphic Deformations and Topological Changes for Trees of 3D Curves

Pierre Louis Antonsanti
Université de Paris - GE Healthcare
pierrelouis.antonsanti@ge.com

The study of tree structures has many applications in for instance the evolution of species (phylogenetic trees), biological structures (plants) or anatomical ones (vascular or pulmonary trees). When labeling the extremities of these trees, the leaves, we face a type of topological change related to the order of appearance of the different branches in the trees. In the presented work we propose a framework for the alignment of 3D curve trees allowing both geometrical and topological deformations of an object, the source, onto a target. For this end we combine the tree space representation proposed by [1] by immersing the trees in a space composed of juxtaposed Euclidean spaces called orthant with a LDDMM deformation guided by Optimal Transport such as [2]. Each orthant is associated with a given topology found in a database or built from a priori knowledge. The trees are considered *rooted* and binary, and one topology corresponds to a unique bifurcation ordering. The source tree of our registration problem is moving inside the tree space.

The LDDMM then provide a realistic diffeomorphic deformation of the shape along with its ambient space. The Optimal Transport is used in the data attachment term to enforce a pairing of the branches from the deformed source and the target. The overall registration is formulated as the minimization of a cost function, sum of the classic LDDMM cost function and a regularization term in the tree space.



(a) (b) (c) (d)

Figure 1: Registration of a template tree (a) obtained with the Sturm Mean in the space of tree-like shapes obtained with 20 trees and 11 possible topologies. The intermediate registration at coarse scale of the data attachment term in the optimization (b) seeks for the correct topology. The registration at the end of the minimization procedure (c) is aligned to the target's geometry and topology (d).

Joint work with: Joan Glaunès.

References

[1] A. Feragen. Toward a theory of statistical tree-shape analysis. *IEEE transactions on pattern analysis and machine intelligence*, 35(8):2008–2021, 2012.

[2] J. Feydy. Optimal Transport for Diffeomorphic Registration. *Lecture Notes in Computer Science*, 291–299, 2017.