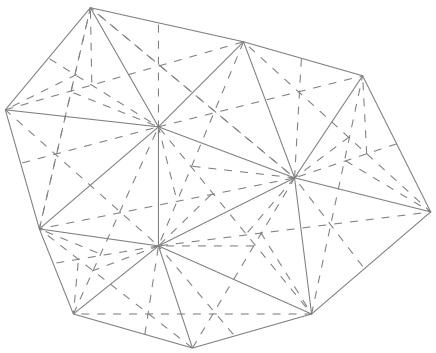
C^2 quartic splines on mixed macro-structures

Salah Eddargani
MISI Laboratory, Hassan First University of Settat, Settat 26000, Morocco
s.eddargani@gmail.com

The polynomial spline functions defined on triangulations are tools widely used in many different fields, both theoretical and applied [1]. It is well known that C^r-regularity of a spline on a given triangulation is obtained if all derivatives up to order $2r$ at the vertices of the triangles, in which case the degree must be greater than or equal to $4r + 1$ [2]. As in practice it is essential to use splines of the lowest degree for a given class, different finite elements obtained by subdividing every triangle have been introduced and analysed in the literature, among them the Clough-Tocher (3-CT), Powell-Sabin (6-PS) and Morgan-Scott (MS-) splits [3, 4, 5], so that C^2 smoothness results, for minimum degrees 6, 5 and 5, respectively. The construction of C^2-continuous quartic splines on a triangulation endowed with a mixed split consisting of macro-triangles with PS-6 or Modified Morgan-Scott (MMS-10) refinements is addressed. Indeed, in [5, 6] it is proved that under a certain geometrical conditions between macro-triangles and edge split points, the space of almost C^2-continuous splines introduced in [7] becomes a subspace of the space of C^2-continuous functions. Joining the opposite vertices of every two triangles sharing an edge gives, in general, a mixed-type triangulation in the above sense. This procedure may result in a PS-6 refinement or an MS-split, from which an MMS-10 split is easily obtained.



For the mixed-type sub-triangulation, the construction of a basis of B-spline-like functions will be provided to establish a suitable representation of the C^2-continuous functions of the space.

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