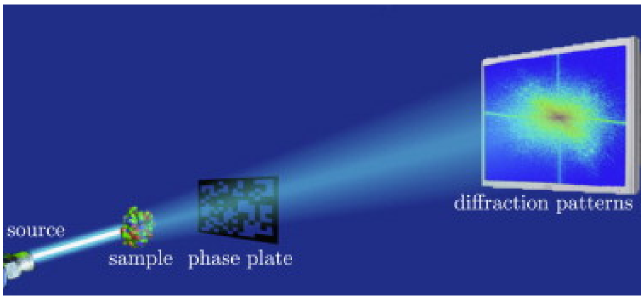
Uniqueness of phase retrieval from three measurements

Philippe Jaming
IMB-Université de Bordeaux
philippe.jaming@math.u-bordeaux.fr

The phase retrieval in diffraction theory mainly consists in recovering a signal $f \in L^2(\mathbb{R}^d)$ from the modulus of its Fourier transform $|\widehat{f}|$. It is well known that this problem is severelly ill posed. One of the reasons for this is that the solution set $\{g \in L^2(\mathbb{R}^d) : |\widehat{g}| = |\widehat{f}|\}$ may be very large.

One may thus try to reduce the solution set by adding more measurements. For instance, one may add a mask which amounts to replacing the measurement $|\widehat{f}|$ by $|\widehat{mf}|$ where m is a masking function. A good description of this idea and actual experiments which are done this way can be found in [1] where the data $|\widehat{mf}|$ has been called a *coded diffraction patern*. The experiments may be summarized in the following picture taken from [1].



So far this problem has mainly been studied in the discrete setting, that is, with $L^2(\mathbb{R}^d)$ replaced by $L^2\big((\mathbb{Z}/N\mathbb{Z})^d\big)$, the Fourier transform being then the discrete Fourier transform. It is then known that 3 well chosen masks can determine almost all signals (but not all) while a small number of randomly chosen masks allow to determine all signals.

The aim of this talk is to show that the situation is better in the continuous setting:

Main result. *Let* $\gamma_1(t) = e^{-\pi t^2}$, $\gamma_2(t) = te^{-t^2/2}$ $\gamma_2(t) := 2\pi t\gamma(t)$, $\gamma_3(t) := (1 - 2\pi t)\gamma(t)$. *Let* $f, g \in L^2(\mathbb{R})$ *be such that* $|\widehat{f\gamma_j}| = |\widehat{g\gamma_j}|$ *then there exists* $c \in \mathbb{C}$ *with* $|c| = 1$ *such that* $f = cg$.

The proof uses complex analysis and the original idea comes from a paper by McDonald [2]. A stronger argument allows to construct more families of masks that lead to the same result. We will also explain that the direct analogues of these masks in the discrete setting don't work and that this is mainly an issue about under-sampling.

Joint work with: Martin Rathmair (Bordeaux)

References

[1] E.J. Candes, Y.C. Eldar, T. Strohmer, V. Voroninski Phase retrieval via matrix completion *SIAM J. Imaging Sci.*, 6(1):199–225, 2013.

[2] J. Mc Donald Phase retrieval and magnitude retrieval of entire functions *J. Fourier Anal. Appl.*, 10:259–267, 2004.