Surfaces with polynomial area element and related topics

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Surfaces possessing Pythagorean normal vector fields (PN surfaces) were introduced in [8] as surface counterparts to the Pythagorean hodograph (PH) curves defined in [4]. PN surfaces have rational offsets and thus provide an elegant solution to many offset-based problems occurring in various practical applications. When we use as the defining property of these 2-surfaces in the Euclidean space $\mathbb{R}^3$ that they possess a polynomial/rational area element we may extend the study to higher dimensions and consider also non-Euclidean metrics. For instance in the Minkowski space $\mathbb{R}^{3,1}$ we obtain the so called MOS surfaces, see [5]. In addition, this approach better captures the analogy with the PH curves which have a polynomial/rational length element, cf. [3].

It is interesting to study these objects both from a theoretical point of view and from the point of view of applications (for instance with emphasis on interpolation and approximation techniques). It is also a challenge to find which well known surfaces fall into the distinguished classes with Pythagorean property. We present some examples of these shapes, some construction algorithms, their application in geometric modelling, categorize recent results into a number of broad themes, and we also mention some open question in this area.

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