Approximate C^1-smoothness for isogeometric analysis over multi-patch domains

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In this talk we study discretization spaces over multi-patch domains, which can be used for isogeometric analysis of fourth order partial differential equations (PDEs). While standard Galerkin discretization of fourth order PDE problems require C^1-smooth discretizations, we propose a method that uses approximate C^1-smoothness, cf. [2, 4].

A key property of IGA is that it is simple to achieve high order smoothness within a single tensor-product B-spline (or NURBS) patch. However, to increase the geometric flexibility, one has to construct spaces beyond such a tensor-product structure. This can be done using unstructured splines, e.g., as in [3], or using a multi-patch construction. While C^0-matching multi-patch domains are easy to construct, C^1-smoothness is harder to achieve. It was shown in [1], that C^1-smooth isogeometric discretizations over G^1-smooth multi-patch domains do in general not possess sufficient approximation power. This issue was circumvented in [1] by restricting to a smaller class of G^1-smooth multi-patch parametrizations, so called analysis-suitable G^1 multi-patch parametrizations, which yield C^1-smooth isogeometric spaces. However, to avoid this additional restriction, we relax the smoothness constraints and construct isogeometric spaces that yield optimal convergence rates in numerical experiments while being only approximately C^1.

Such constructions are of interest when solving numerically fourth-order problems, such as the biharmonic equation or Kirchhoff-Love plate and shell formulations. The approximate C^1 method is advantageous when compared to alternatives that rely on a weak imposition of smoothness, such as Nitsche's method. In contrast to weakly imposing coupling conditions, the approximate C^1 construction is explicit and no additional terms need to be introduced to penalize the jump of the derivative at the interface. Thus, the approximate C^1 method can be used more easily as no additional parameters need to be estimated.

Joint work with: Pascal Weinmüller.

References

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