Polynomiality vs. rationality of Pythagorean hodograph/normal curves and surfaces

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We will discuss the relation between the polynomial and rational curves with pythagorean hodograph in $\mathbb{R}^2$ and $\mathbb{R}^3$ as well as the rational and polynomial pythagorean normal surfaces in $\mathbb{R}^3$.

The planar cases are considered rather for the seek of completeness and as a motivation. Indeed the relation between the planar polynomial and rational PH curves was already fully analyzed in [3]. We will however compare these two families of curves using a different method based on solving a system of linear equations.

The situation is much more interesting in $\mathbb{R}^3$. Historically the polynomial PH curves [1] are much better studied then the rational ones, [4, 8]. On the other hand the rational PN surfaces were fully described already in [2] but only examples of polynomial PN surfaces are available, see e.g. [5, 6]

We propose a new method for studying these problems. It is based on determining the corresponding motion polynomial, [7, 9]. While the primal (rotation) component of the motion polynomial is arbitrary, the dual (translation) part is determined be a linear system of equations. This system is analysed and possible denominators of the resulting PH/PN curves and surfaces are discussed. Polynomial object in this approach appear as special cases of the polynomial ones. From a certain point of view however the polynomial objects appear to be the generic cases.

Joint work with: Hans-Peter Schröcker, Daniel Scharler, Bahar Kalkan.

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