From (β, γ)-Chebyshev functions of the interval to (β, γ)-Lissajous curves of the square

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Chebyshev polynomials are a classical topic in scientific literature, and they have been considered in many fields of research. For example, the related zeros are particularly suitable for polynomial interpolation on the interval $[-1, 1]$ due to their well conditioning. Moreover, the extrema of Chebyshev polynomials, along with the set $\{-1, 1\}$, form the set of *Chebyshev-Lobatto* (CL) points, which are *quasi-optimal* interpolation nodes as well [1, 2]. In [3], we introduced and analysed a new class of (β, γ)-Chebyshev functions and points, which can be seen as a generalisation of classical Chebyshev polynomials and points (see Figure 1). The achieved theoretical findings have been employed in [4] for reducing the effects of both Runge's and Gibbs phenomena, in the framework of the *fake nodes approach* [5].

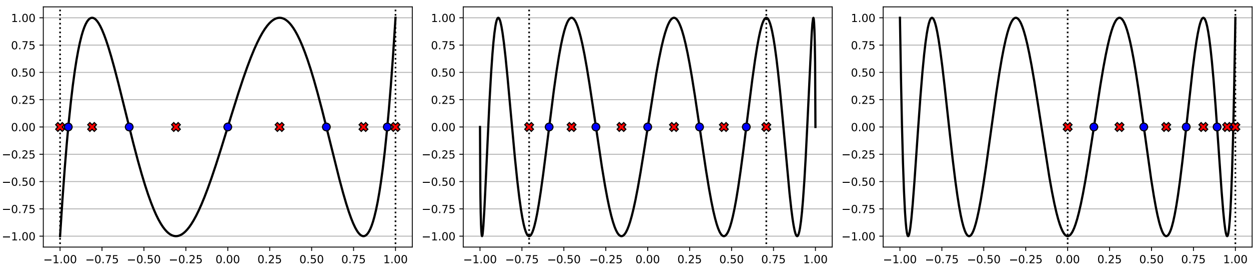

Figure 1: Left: an example of Chebyshev polynomial (solid line), Chebyshev points of the first kind (blue circles) and CL points (red crosses). Centre and right: two examples of (β, γ)-Chebyshev functions (solid line), (β, γ)-Chebyshev points (blue circles) and (β, γ)-CL points (red crosses). Depending on the choice of the parameters, they can be symmetric or not with respect to the origin.

In the square $[-1, 1]^2$, unions of tensor-product Chebyshev grids provide sets of nodes that guarantee a stable polynomial interpolation process and that can be characterised as self-intersection or square-tangency points of Lissajous curves [6]. This paves the way for the study of (β, γ)-Chebyshev grids and for the analysis of polynomial approximation schemes along (β, γ)-Lissajous curves in $[-1, 1]^2$, in view of designing a unified generalised framework.

Joint work with: Stefano De Marchi, Giacomo Elefante.

References

[1] T.J. Rivlin An Introduction to the Approximation of Functions. *Dover Publications*, New York, 1969.

[2] J.C. Mason, D.C. Handscomb Chebyshev Polynomials (1st ed.). *Chapman and Hall/CRC*, 2002.

[3] S. De Marchi, G. Elefante, F. Marchetti. On (β, γ)-Chebyshev functions and points of the interval. *J. Approx. Theory*, 105634, 2021.

[4] S. De Marchi, G. Elefante, F. Marchetti. Stable discontinuous mapped bases: the Gibbs-Runge-Avoiding Stable Polynomial Approximation (GRASPA) method. *Comp. Appl. Math.*, 40:299, 2021.

[5] S. De Marchi, F. Marchetti, E. Perracchione, D. Poggiali. Polynomial interpolation via mapped bases without resampling. *J. Comput. Appl. Math.*, 364:112347, 2020.

[6] W. Erb, C. Kaethner, P. Dencker, M. Ahlborg. A survey on bivariate Lagrange interpolation on Lissajous nodes. *Dolom. Res. Notes on Approx.*, 8:23–36, 2015.