Maximum relative distance between real rank-two and rank-one tensors

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We investigate the maximum distance of a rank-two tensor to rank-one tensors. An equivalent problem is given by the minimal ratio of spectral and Frobenius norm of a tensor. For matrices the distance of a rank k matrix to a rank r matrices is determined by its singular values, but since there is a lack of a fitting analog of the singular value decomposition for tensors, this question is more difficult in the regime of tensors. We extend the results in [1] and show that the distance of a rank-two tensor $\mathbf{A}$ of order d to the set of rank-one tensors is bounded by

$$\min_{\operatorname{rank}\mathbf{B}=1} \|\mathbf{A}-\mathbf{B}\|_{\mathsf{F}} < \sqrt{1-\left(1-\frac{1}{d}\right)^{d-1}}\,\|\mathbf{A}\|_{\mathsf{F}},$$

where $\|\cdot\|_{\mathsf{F}}$ is the Frobenius norm. It is in particular remarkable that the constant in the right-hand side is uniformly bounded by $\sqrt{1-\left(1-\frac{1}{d}\right)^{d-1}} < \sqrt{1-\frac{1}{e}}$ and is again sharp for $d \to \infty$. We therefore have a bound for the distance of a rank-two tensor of any order to the set of rank-one tensors.

Joint work with: André Uschmajew.

References

[1] H. Eisenmann and A. Uschmajew. Maximum relative distance between symmetric rank-two and rank-one tensors. *ArXiv:*2111.12611, 2021.