Geometric Hermite interpolation in R^n

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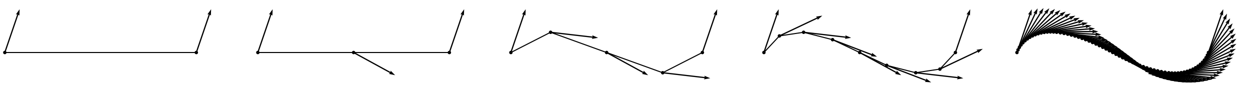
The geometric Hermite data consists of point samples and their associated tangent vectors of unit length. Thus, the geometric Hermite interpolation problem is to interpolate such data. Extending the classical Hermite interpolation of functions, this geometric Hermite problem has become popular in recent years and has ignited a series of solutions in the 2D plane and 3D space such as in [1, 2, 4, 5, 6]. We propose a general approach for constructing a broad class of operators approximating high-dimensional curves, which are based on geometric Hermite data. In particular, we present a method for approximating curves that is valid in any dimension.

As the basic building block, we pose the notion of Hermite average. In my talk, I will present a formal definition of the latter, while addressing some of the fundamental challenges accompanying such an attempt. I will introduce the Bézier average which is an example of a Hermite average, and demonstrate how we use it to modify subdivision schemes which are based on repeated averages. The special case of interpolating Hermite data by repeatedly refining it with Hermite averages, is proved to converge. Furthermore, its limit inherits geometric properties of the average, such as circle preserving. We will address the latter and discuss the key arguments in the proof.

We will also consider some properties of approximation. While subdivision schemes commonly achieve their full approximation power under restricted assumptions about the sampling method as investigated in [3], our result is an interpolatory approximation which is robust to sampling, that is, it does not depend on a particular sampling policy. In fact, it is implied numerically that refining data by Bézier average yields a fourth order approximation. This, as well as other numerical examples, elucidate the advantages of our approach. Finally, we observe that such an approach naturally extends to a more general setting of approximating manifold valued curves by a suitable adjustment of the Hermite average.

The figure below presents the process of refining two given Hermite samples by inserting their Bézier average.



Joint work with: Nira Dyn, Nir Sharon.

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