Analyticity and sparsity in uncertainty quantification for PDEs with Gaussian random field inputs

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We establish summability results for coefficient sequences of Wiener-Hermite polynomial chaos expansions for countably-parametric solutions of linear elliptic and parabolic divergence-form partial differential equations with Gaussian random field inputs. The proof is based on analytic continuation of parametric solutions into the complex domain. This holomorphy-based argument yields a "differentiation-free" sparsity analysis in various scales of function spaces. It also applies to certain posterior densities in Bayesian inverse problems subject to Gaussian priors on uncertain inputs from function spaces. This allows us to prove dimension-independent convergence rates of various constructive high-dimensional deterministic numerical approximation schemes such as single-level and multi-level versions of anisotropic sparse-grid Hermite-Smolyak interpolation and quadrature in both forward and inverse computational uncertainty quantification. Finally we discuss some implications for neural network approximation.

Joint work with: Dinh Dũng, Van Kien Nguyen, Christoph Schwab

References

[1] Dinh Dũng, Van Kien Nguyen, Christoph Schwab and Jakob Zech. Analyticity and sparsity in uncertainty quantification for PDEs with Gaussian random field inputs. arXiv:2201.01912, 2022.

[2] Deep Learning in High Dimension: Neural Network Approximation of Analytic Functions in $L^2(\mathbb{R}^d, \gamma_d)$. Christoph Schwab and Jakob Zech. arXiv:2111.07080, 2021.