From Delaunay to Curved Optimal Delaunay Triangulations

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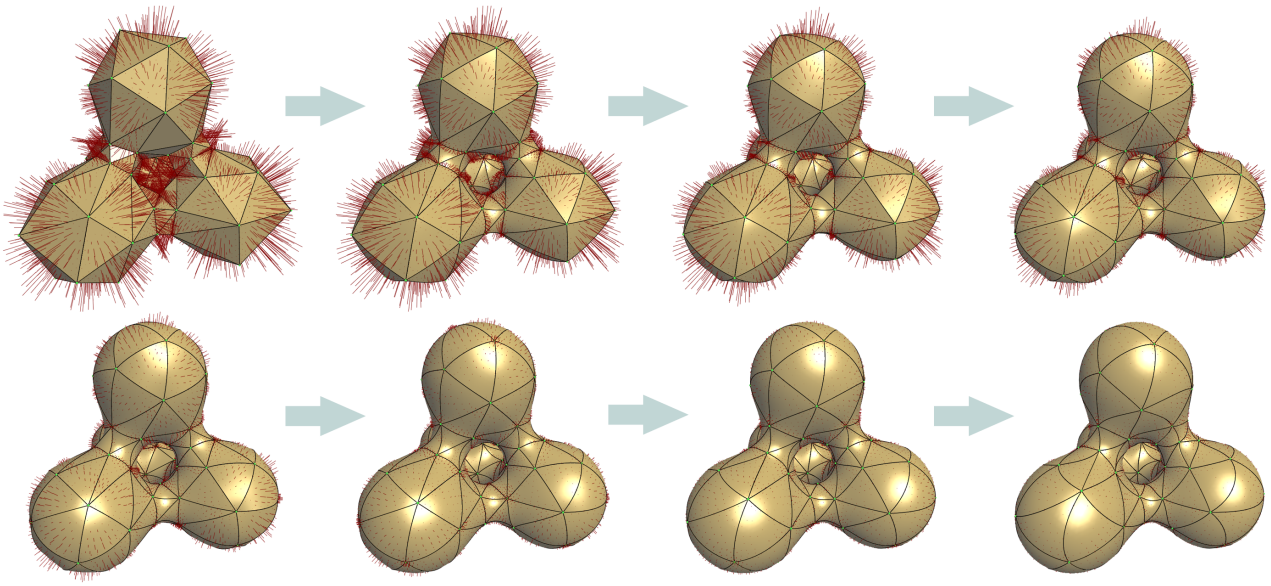

Figure 1: A Bézier mesh (here with 200 cubic patches) can capture a curved domain with orders of magnitude less elements than a linear counterpart, for a given Hausdorff distance.

Meshes with curvilinear elements hold the appealing promise of higher-order numerical accuracy compared to their commonly-used straight-edge counterparts. However, the generation of curved meshes remains a computationally expensive endeavor: high-order parametric elements are notoriously difficult to conform to a given boundary geometry, and enforcing a smooth and non-degenerate Jacobian everywhere brings additional numerical difficulties to the meshing of complex domains. We extend Optimal Delaunay Triangulations (ODT) [1] to curved and graded isotropic meshes. We show that the measure of element distortion underlying the ODT approach can be re-expressed as a potential energy whose minimization amounts to an equidistribution of the gradient of the deformation field, thus regularizing simultaneously the size and shape of the simplicial elements. After formulating a non-shrinking traction to favor uniform and isotropic elements at the boundary, we show that this interpretation of ODT also applies for curved meshes made of Bézier simplices. The resulting curved meshes provide coarse geometric descriptions of arbitrary 2D or 3D domains with a much improved fit to the domain boundary due to their piecewise polynomial nature, see Figure 1. Moreover, our construction naturally promotes smoothness of the gradient of the induced geometric map inside and across elements [2].

Joint work with: Leman Feng, Laurent Busé, Hervé Delingette and Mathieu Desbrun.

References

[1] Long Chen, Jin-chao Xu. Optimal Delaunay Triangulations. *J. Computational Mathematics*, 299–308, 2004.

[2] Leman Feng, Pierre Alliez, Laurent Busé, Hervé Delingette and Mathieu Desbrun. Curved Optimal Delaunay Triangulation. *ACM Transactions on Graphics - Proceedings of SIGGRAPH*, 2018.