Topologically Unrestricted Isogeometric Splines on Multi-Patch Domains with Extraordinary Vertices

Philipp Langgruber
Johannes Kepler University, Linz/Austria
philipp.langgruber@jku.at

Isogeometric splines on multipatch domains (cf. [1]) are needed for the discretization of partial differential equations on general domains. However, achieving smoothness at extraordinary vertices (EVs) – while maintaining the approximation power – is a challenging problem. Several approaches for solving this problem have been explored in the rich literature on this topic. These may be classified into methods that (a) use the classical notion of geometric continuity between surface patches (e.g., [3]), (b) employ singularly parameterized surfaces (e.g., [2, 4]), (c) rely on subdivision surfaces (e.g., [5]) and (d) are based on the concept of manifolds from differential geometry (e.g. [6]). Our work aims at applications in isogeometric analysis, where it is essential to use discretization spaces spanned by relatively simple basis functions (suitable for efficient matrix assembly via numerical integration), which also possess good approximation power, ideally guaranteed by theoretical results. We focus on a construction of Prautzsch [7], which is based on composing polynomial mappings with spline parameterizations. We show how to generate suitable basis functions in the vicinity of EVs and discuss the approximation power of the resulting spline space.

Joint work with: Bert Jüttler

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