Surfaces with a constant ratio of principal curvatures

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Motivated by applications in architectural geometry [1], we study and compute surfaces with a constant ratio $a = \kappa_1/\kappa_2$ of principal curvatures (CRPC surfaces). While the special case of minimal surfaces ($a = -1$) is very well understood and has been studied in great detail, almost nothing is known about general CRPC surfaces.

Our computational approach is based on discrete differential geometry and numerical optimization [2, 3]. We discretize the so-called characteristic parameterizations of CRPC surfaces. For negative Gaussian curvature K, these parameterizations are asymptotic. For positive K they are conjugate and symmetric with respect to the principal curvature directions. CRPC surfaces are characterized by a constant angle between the parameter lines of the characteristic parameterization.

The developed computational methods also serve as an experimental basis for mathematical studies of the largely unexplored class of CRPC surfaces. Some phenomena suggested from computational results have already been verified by mathematical analysis. This concerns the classes of helical and spiral CRPC surfaces, for which we can provide explicit parameterizations and a study of the arising shapes.

Joint work with: Yang Liu, Olimjoni Pirahmad, Hui Wang.

References

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