Learning Scale Invariant Signatures for Planar Curves

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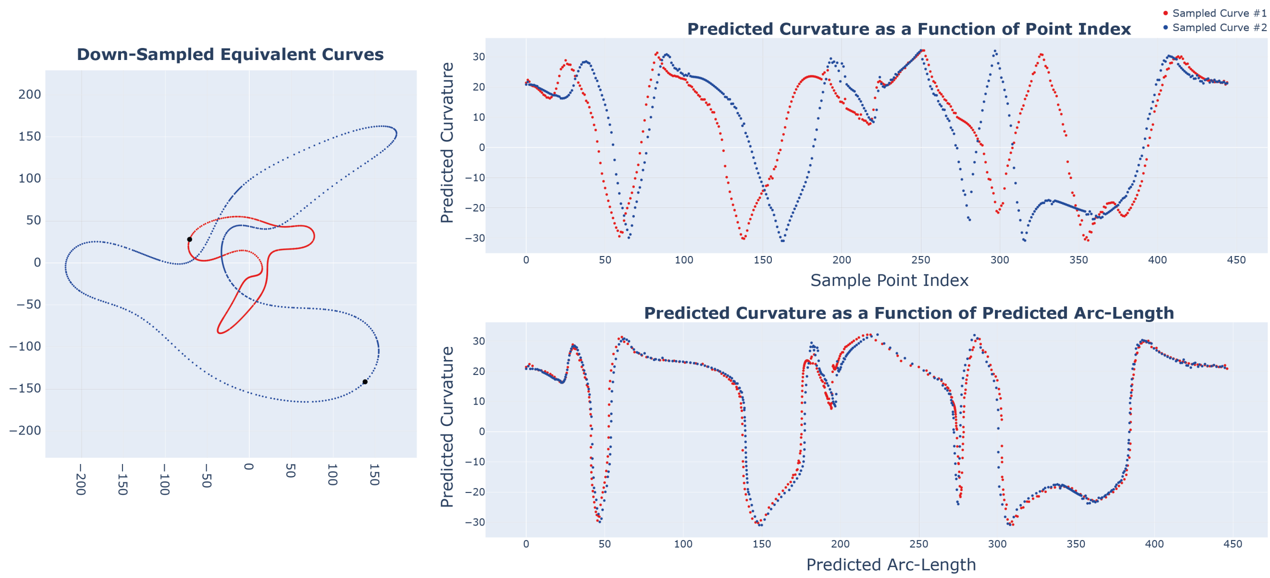
An important theorem by É. Cartan [2, p. 183, Thrm. 8.53] states that two curves are related by a group of transformation iff their signature curves w.r.t. the transformation group are identical. Signature curves are defined by differential invariants, such signatures of planar curves involve the evaluation of invariant differential measures. In practice, planar curves are digitally represented as a discrete set of points, which implies that the computation of differential invariant quantities, such as the group-curvature at a point, can only be numerically approximated using finite differences techniques. Since differential invariants often involve high-order derivatives, their approximations are prone to numerical instabilities due to sensitivity to sampling noise.

Here, we focus on the similarity group of transformations. Also known as *uniform scaling*, similarity transforms introduce numerical challenges for constructing reparametrization-invariant analysis procedures. E.g., similarity-invariant heat flow of planar curves, traditionally used to produce multi-scale representations, is an unstable operation in the uniform scaling case [1]. Motivated by the above challenges, we propose a novel deep-learning approach, to produce numerically-stable and reparametrization-invariant approximation models for the differential invariants of planar curves w.r.t. the similarity group. We qualitatively evaluate our results by plotting matching invariant signatures for equivalent curves w.r.t. the similarity group.

The following figure presents an example of similarity-invariant signature-curves estimated by our learning models. **Left**: Two equivalent curves w.r.t. the similarity group, both sampled non-uniformly. The black points define corresponding reference points. Curve traversal is done in clock-wise fashion. **Top-Right**: Our learned group-curvature at each point as a function of the point indices. The two plots do not align since the two curves were sampled differently. **Bottom-Right**: Our learned group-curvature at each point as a function of our learned group arc-length. When learning the group arc length, these new signature graphs do align.



Joint work with: Ron Kimmel, Technion.

References

[1] Alfred Bruckstein and Doron Shaked. *On Projective Invariant Smoothing and Evolutions of Planar Curves and Polygons*. Vol. 7. June 1997, pp. 225–240. DOI: 10.1023/A:1008226427785.

[2] Peter J. Olver. *Classical Invariant Theory*. London Mathematical Society Student Texts. Cambridge University Press, 1999. DOI: 10.1017/CBO9780511623660.