Cardinal and semi-cardinal interpolation with Matérn kernels

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Let $\phi : \mathbb{R}^d \to \mathbb{R}$ be a continuous and symmetric function (i.e. $\phi(-x) = \phi(x)$, $x \in \mathbb{R}^d$), satisfying a suitable decay condition for large $\|x\|$, and define the shift-invariant space

$$S(\phi) = \{ \sum_{k \in \mathbb{Z}^d} c_k \phi(\cdot - k) : (c_k) \in \ell^\infty(\mathbb{Z}^d) \}.$$

The problem of *cardinal interpolation* with the kernel ϕ is to find, for some data $(y_j)_{j \in \mathbb{Z}^d}$, a function $s \in S(\phi)$, such that $s(j) = y_j$, for all $j \in \mathbb{Z}^d$. If this problem admits a unique solution for any bounded data, it is known that specific algebraic or exponential decay of the kernel ϕ is transferred to the corresponding *Lagrange function* for cardinal interpolation on $\mathbb{Z}^d$, leading to a well-localized Lagrange representation of the solution s.

The first part of the talk presents a similar result obtained for the related problem of *semi-cardinal interpolation*, in which the multi-integer grid $\mathbb{Z}^d$ is replaced by a half-space lattice $H \subset \mathbb{Z}^d$. Despite the loss of shift-invariance in this case, we prove that the algebraic or exponential decay still carries over from ϕ to the corresponding H-indexed family of semi-cardinal Lagrange functions, with constants that are independent of the index $j \in H$ (see [1]).

In the second part, we discuss two recent applications and refinements of the above results for the Matérn kernel $\phi := \phi_{m,d}$, defined, for a positive integer $m > d/2$, as the exponentially decaying fundamental solution of the elliptic operator $(1 - \Delta)^m$ in $\mathbb{R}^d$, where Δ is the Laplace operator. Namely, for a scaling parameter $h > 0$, we consider non-stationary interpolation to data prescribed on $h\mathbb{Z}^d$ from the *flat ladder* collection $\{S_h(\phi)\}_h$ generated by ϕ via

$$S_h(\phi) = \{ \sum_{k \in \mathbb{Z}^d} c_k \phi(\cdot - hk) : (c_k) \in \ell^\infty(\mathbb{Z}^d) \}.$$

For this problem, we prove that the Lebesgue constant of the associated interpolation operator is uniformly bounded as $h \to 0$, which allows us to deduce the maximal L^∞-convergence rate $O(h^{2m})$ for the Matérn flat ladder interpolation scheme (see [2]). On the other hand, if $d = 1$, then the translates of the Matérn kernel $\phi_{m,1}$ span a linear space of exponential splines, for which we show that non-stationary semi-cardinal interpolation on the scaled grid $h\mathbb{Z}_+$ achieves the convergence rate $O(h^m)$ in $L^\infty(\mathbb{R}_+)$, amounting to half of the approximation order of the corresponding cardinal scaled scheme.

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References

[1] A. Bejancu. Wiener-Hopf difference equations and semi-cardinal interpolation with integrable convolution kernels. *Preprint*, arXiv:2006.05282, 2020.

[2] A. Bejancu. Uniformly bounded Lebesgue constants for scaled cardinal interpolation with Matérn kernels. *Preprint*, arXiv:2009.00711, 2020.