Global constraints in Hermite interpolation problems

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Hermite interpolation of discrete data — points, tangents, curvatures, etc. — is a common approach to the construction of planar and spatial curves. The imposition of global (integral) constraints is more difficult, and therefore less commonly considered. We consider two types of global constraints that can be exactly achieved by using Pythagorean–hodograph curves. The first is the imposition of an exact arc length for the interpolant, and it is shown that this can be achieved for both for planar and spatial G^1 end–point data by use of quintic Pythagorean–hodograph curves [1, 2]. The second constraint involves the construction of a rational adapted orthonormal frame (comprising the curve tangent and two unit vectors spanning the curve normal plane) that satisfies prescribed initial/final orientations. Since the well–known rotation–minimizing frames are solutions of an initial–value problem, they are incompatible with this constraint. Consequently, the *minimal–twist frame* is introduced — an orthonormal frame with prescribed initial and final instances, with the least possible value for the integral of the tangent component of its angular velocity. The construction of rational minimal twist frames on both open and smooth closed–loop Pythagorean–hodograph curves is demonstrated [3, 4].

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References

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