G^1-smooth Biquintic Approximation of Catmull-Clark Subdivision Surfaces

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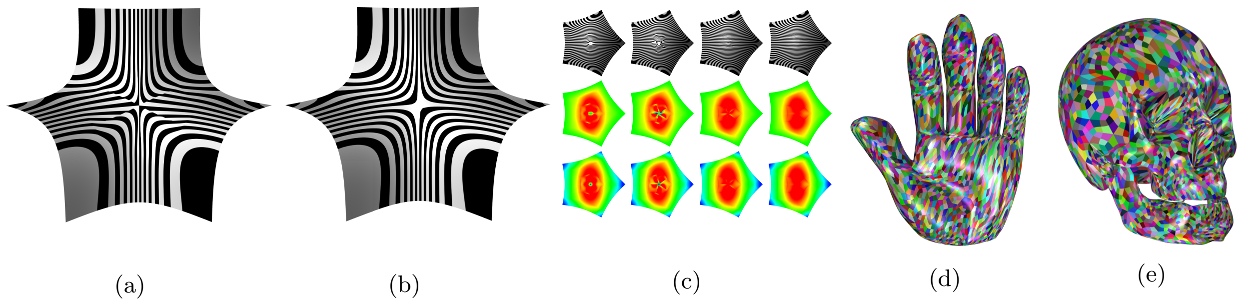

Figure 1: (a): isophotes on ACC_3 surface. (b): isophotes on our G^1 surface. (c): isophotes and curvature analysis for each of the four solving strategies. (d)-(e): surfaces from complex meshes with many of EVs.

In this work a new construction of a globally G^1 smooth family of Bézier surfaces, defined by smoothing masks approximating the well-known Catmull-Clark (CC) subdivision surface, is presented. The resulting surface is a collection of Bézier (quad) patches, which are bicubic C^2 around regular vertices and biquintic G^1 around Extraordinary Vertices (EVs) i.e., in our case, vertices with valence $N \neq 4$. Starting from the work of Loop and Shaefer [1], which provides a bicubic approximation of the CC limit surface with only C^0 regularity around EVs, we improve this construction to reach a surface with global G^1 smoothness. From the bicubic case, applying twice the classical degree elevation algorithm for Bézier surfaces, we achieve enough degree of freedom to impose G^1 conditions between adjacent patches: those conditions are assigned making use of quadratic gluing data functions [2] around EVs which depends just on their valence. This construction leads to a linear system of seven equations per interior edge to be solved: each equation involves smoothing masks of symmetric control points with respect to the edge. In the case of inner EVs, i.e. not lying on a boundary, some equations lead to (degenerate) circulant system to be solved and some return direct constrains: in all cases, we are able to solve in an explicit way these relations. We present explicit formulas for G^1 smoothing masks; moreover, these solutions possess degrees of freedom which can be fixed arbitrary. The entire system presents more than a way to be solved, and this yields a family of G^1 solutions; between all the possible solving strategies, we identify four of them and we analyze each to determine the best solving strategies returning the smoothest surface. In order to assert the quality of the resulting surfaces and identify the ones that lead to the best result, both visually and numerically, we conduct curvature analysis on an extensive benchmark of meshes with different features. We also treat the case of EVs on boundaries; in this setting, we keep unaltered the boundary masks for the ACC_3 surface presented in [1] and we force the inner edges to satisfy the G^1 relations. Similarly to the inner case, this construction leads to explicit formulas for G^1 smoothing masks.

The resulting construction is described by explicit masks applied to the input control mesh, providing efficient computation and fast rendering of smooth piecewise polynomial surfaces of low degree and arbitrary topology.

Joint work with: Angelos Mantzaflaris, Bernard Mourrain.

References

[1] C. Loop, S. Shaefer. Approximating Catmull-Clark subdivision surfaces with bicubic patches. *ACM Transactions on Graphics*, 10.1145/1330511.1330519, 2008.

[2] A. Blidia, B. Mourrain, N. Villamizar. G1-smooth splines on quad meshes with 4-split macro-patch elements. *Computer Aided Geometric Design*, 10.1016/j.cagd.2017.03.003, 2017.