Weighted least-squares approximation in expected L^2 norm

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We investigate the problem of approximating a function u in L^2 with a linear space of functions of dimension n, using only evaluations of u at m chosen points, with m of the order of n. A first approach [2], based on weighted least-squares at i.i.d random points, provides a near-best approximation of u, but requires m of order $n \log(n)$. To reduce the sample size while preserving the quality of approximation, we need a result on sums of rank-one matrices from [3], which answers to the Kadison-Singer conjecture. The results presented here, expressed in expected L^2 norm of the approximation error, can be found in [1] and will be compared to alternative approaches [4, 5].

Joint work with: Albert Cohen.

References

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