On the numerical stability of barycentric rational interpolation

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The barycentric forms of polynomial and rational interpolation have recently gained popularity, because they can be computed with simple, efficient, and numerically stable algorithms [1, 2, 3]. In this talk, we show more generally that the evaluation of any function that can be expressed as $r(x) = \sum_{i=0}^{n} a_i(x) f_i \big/ \sum_{j=0}^{m} b_j(x)$ in terms of data values f_i and some functions a_i and b_j for $i = 0, \ldots, n$ and $j = 0, \ldots, m$ with a simple algorithm that first sums up the terms in the numerator and the denominator, followed by a final division, is forward and backward stable under certain assumptions. This result includes the two barycentric forms of rational interpolation as special cases. Our analysis further reveals that the stability of the second barycentric form depends on the Lebesgue constant associated with the interpolation nodes, which typically grows with n, whereas the stability of the first barycentric form depends on a similar, but different quantity, that can be bounded in terms of the mesh ratio, regardless of n. We support our theoretical results with numerical experiments, which indicate that the first barycentric form is stable even in situations where the second barycentric form is completely unstable, as shown in Figure 1.

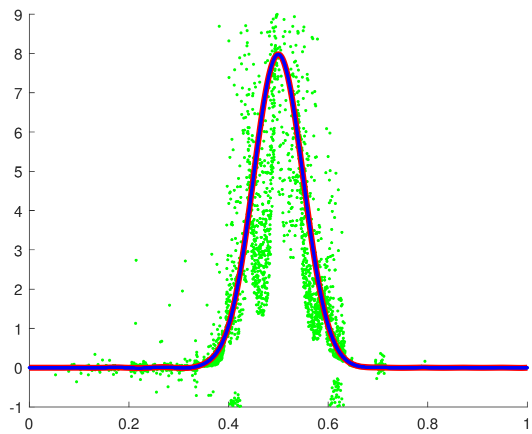

Figure 1: Multiple-precision implementation of a rational barycentric interpolant $r(x)$ for $x \in [0, 1]$ (red) and numerical approximation $\hat{r}(x)$ in double precision for $10,000$ equidistant evaluation points in $[10^3\varepsilon, 1 - 10^3\varepsilon]$, where ε is the machine epsilon, using the standard implementations of the first (blue dots) and the second (green dots) barycentric form.

Joint work with: Kai Hormann, Rosanna Campagna.

References

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