Singular cases of planar and spatial C^1 Hermite interpolation problems based on quintic Pythagorean-hodograph curves

Kai Hormann
Università della Svizzera italiana
kai.hormann@usi.ch

A well-known feature of the Pythagorean-hodograph (PH) curves is the multiplicity of solutions arising from their construction through the interpolation of Hermite data. In general, there are four distinct planar quintic PH curves that match first-order Hermite data [1], and a two-parameter family of spatial quintic PH curves compatible with such data [2]. Under certain special circumstances, however, the number of distinct solutions is reduced [3]. Specifically, in the planar case it turns out that there may be only three (but not less) distinct Hermite interpolants, of which one is a "double" solution (see Fig. 1). In the spatial case, a constant difference between the two free parameters reduces the dimension of the solution set from two to one, resulting in a family of quintic PH space curves of different shape but identical arc lengths. The values of the free parameters that result in formal specialization of the (quaternion) spatial problem to the (complex) planar problem are also identified, demonstrating that the planar PH quintics, including their degenerate cases, are subsumed as a proper subset of the spatial PH quintics.
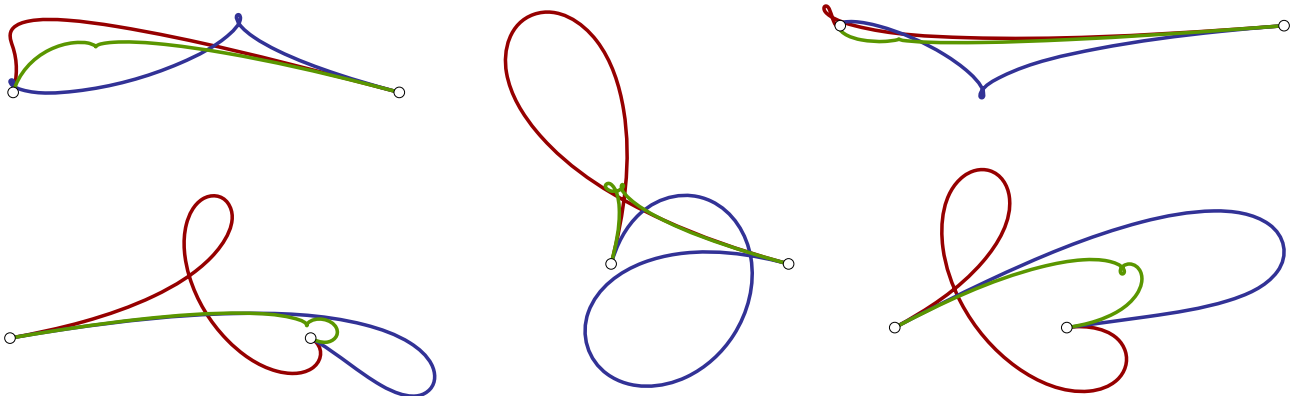
Figure 1: Examples of planar cases with only three distinct PH quintic Hermite interpolants. In each case, the green curve corresponds to the "double" solution.

Joint work with: Rida Farouki, Federico Nudo

References

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[3] R. T. Farouki, K. Hormann, and F. Nudo. Singular cases of planar and spatial C^1 Hermite interpolation problems based on quintic Pythagorean-hodograph curves. *Computer Aided Geometric Design*, 82:Article 101930, 13 pages, Oct. 2020.