Detecting projective equivalences of planar curves birational to elliptic and hyperelliptic curves

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Recognizing projective equivalence, i.e. detecting whether two objects are related by a projectivity, is important in applied fields like Pattern Recognition and Computer Vision. For rational curves the problem is well-understood, and efficient algorithms for detecting projective equivalence are available [3]. However, for other curves the problem is still open. Here we will consider curves birationally equivalent to elliptic and hyperelliptic planar curves, which are not rational, although they can be parametrized by rational functions and square-roots of rational functions. These curves are well-known in Mathematics, both for their rich mathematical properties and for their applications in Cryptography, and appear naturally in several constructions in Computer Aided Geometric Design [2], like offsets of rational curves, bisectors of certain curves or contour curves of rational canal surfaces. Four our purposes, the most useful property of these curves is that elliptic and hyperelliptic curves have a birational model in the plane defined by Weierstrass equations (Weierstrass normal forms), so that plane curves birationally equivalent to elliptic and hyperelliptic curves also inherit a Weierstrass model by composition of the birational mappings. Thus, from an algorithmic point of view, the key idea is to find a corresponding transformation between the Weierstrass forms of the curves, which in the case of elliptic curves can be completely characterized. The results presented in this talk can be found in [1].

Joint work with: Juan Gerardo Alcázar.

References

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